1 Question worth 20%

A simulation requires variates for the random variable $\mathcal{X}$ with the pdf and cdf:

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 \leq x < \frac{1}{2} \\ 0 & \frac{1}{2} \leq x < \frac{3}{4} \\ 2 & \frac{3}{4} \leq x \leq 1 \end{cases}$$

Write the code that returns a variate from variable $\mathcal{X}$.

```
double v()
{
    double u ;
    u = drand48() ;
    if( u < 0.5 )
        return u ;
    else
        return 0.5 + x / 2 ;
}
```

2 Question worth 20%

We are interested in the properties of a random variable $\mathcal{X}$. A simulation study yielded 100 data points describing $\mathcal{X}$. The sample mean ("central tendency") $\widehat{\theta}$ equals 2.5 and the sample variance ("dispersion") S^2 is 4.

A claim is made:

> *The mean of $\mathcal{X}$ is between 2 and 3 with a significance level of 0.01.*

Rewrite this claim so that it matches the simulation results.

$$2.5 - t_{0.005,99}\frac{2}{10} \leq \mu \leq 2.5 + t_{0.005,99}\frac{2}{10}$$

$t_{0.005,99}$ is not in my table but the points presents suggest that its value is a bit less than 2.65. So, the mean of $\mathcal{X}$ is in the interval 2.5 ± 0.53 with significance 0.1.

Why is it reasonable to claim that no data point should have a value exceeding 5.5?

The formula is $2.5 \pm t_{0.005,99} \times 2 \times \sqrt{\frac{101}{100}} \approx 2.5 \pm 5.3$ for a significance of 0.01. But with a significance levels of 0.1 and 0.2 we have intervals 2.5 ± 3.3 and 2.5 ± 2.6 suggesting that the claim can be made “reasonably” with a confidence of about 85%.

3 Question worth 20%

Consider the following 30 numbers:

0.025	0.065	0.090	0.097	0.138	0.140	0.180
0.245	0.255	0.275	0.390	0.480	0.538	0.582
0.611	0.660	0.770	0.825	1.270	2.45	3.150
4.450	6.100	8.250	12.00	15.30	17. 40	18.50
19.55	19.99					

Test the hypothesis that the numbers are uniformly distributed in the interval $[0, 20]$ using the χ^2 test.

4 Question worth 25%

An old Pentium machine executes a machine instruction incorrectly with probability 10^{-5}. What is the probability that a calculation requiring 500,000 instruction produces a correct result?

$$(1 - 10^{-5})^{500000} = e^{-5} = 6.74 \times 10^{-3}$$

Write a continuous simulator modelling the behaviour of the Pentium (the main simulation loop only, please).

```
for( i = 0 ; i < 500000 ; i++ ) {
    if( drand48() < 1e-5 )
        error++ ;
```

5 Question worth 25%

A search engine server is configured in the following way:

1. All the incoming queries come to the Decision Server (D).
2. D forwards each query to one of the 5 Processing Servers (P0–P4). D takes t_D time to choose the Processing Server and forward a query to it.
3. The Processing Server handles the query and sends a reply directly into the Internet. This requires a total time equal to t_P.

t_D is exponentially distributed with a mean of 0.02 second; t_P is exponentially distributed with a mean of 0.1 second.

Write a discrete simulator that models the operation of this search engine (the main simulation loop only, please).

```
while((E = deletemin()) != NULL ) {
   Now = E→Time ;
   switch( T→Type ) {
   case QUERY:
   case DSTART:
   case DDONE:
   case PSTART:
   case PDONE:
   }
}
```

```
case QUERY:
   if( D == IDLE )
      Insert( EQ , DSTART , Now , 0 ) ;
   q++ ;
   if( Now < SIMEND )
      Insert( EQ , QUERY , Now + IA() , 0 ) ;
   break ;
case DSTART:
   D = BUSY ;
   q-- ;
   Insert( EQ , DDONE , Now + Exp( 0.02 ) , 0 ) ;
   break ;
```

```
case DDONE:
    pi = rand() % 5 ;
    P[pi]++ ;
    Insert( EQ , PSTART , Now , pi ) ;
    if( q > 0 )
        Insert( EQ , DSTART , Now , 0 ) ;
    else
        D = IDLE ;
    break ;
case PSTART:
    if( P[E->c] > 0 )
        Insert( EQ , PDONE , Now + Exp( 0.1 ) , E->c ) ;
    break ;
case PDONE:
    P[E->c]-- ;
    Insert( EQ , PSTART , Now , E->c ) ;
```