The Fuzzy Line Between Among and Surround

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Abstract — **Spatial relationships among image objects play a vital role in countless domains of computer vision (*e.g.*, pattern recognition, image understanding, scene description). Some have received considerable attention the last few years (*e.g.*, "to the right of," "above," "to the left of" and "below"), but others have not been the subject of as much investigation. In this paper, we design consistent fuzzy models of three important spatial relationships: "surrounded by," "between" and "among." These models are based on the histogram of forces, which represents the relative position between two objects. Here, force histograms are assimilated to fuzzy sets and processed through their α-cuts. Our goal is to extend the capabilities of a fuzzy system for linguistic scene description introduced in an earlier paper.**

I. INTRODUCTION

Spatial relationships among image objects play a vital role in countless domains of computer vision. Freeman tried to draw up an exhaustive list of "primitive" relationships and came up with 13 names (such as "above", "near", "between", "inside", etc.) [1]. He also proposed the use of fuzzy relations, because "all or nothing" standard mathematical relations are clearly not suited for models of spatial relationships. Freeman's ideas were widely adopted. However, 2D objects were often assimilated to very elementary entities such as a point (centroid) or a (bounding) rectangle. The procedure is practical, but yields poor models because of the loss of a lot of morphological information. Krishnapuram *et al* [2] and Miyajima and Ralescu [3] tackled this problem in parallel and came up with similar propositions. They developed the idea that the relative position between two objects can have a representation of its own and can thus be described in terms other than spatial relationships. However, the representation set out in [3] shows several weaknesses (*e.g.*, requirement for raster data, long processing times, anisotropy). In [4][5], Matsakis and Wendling introduced the histogram of forces. It proved to be a powerful tool, studied for numerous applications (*e.g.*, pattern recognition [6], scene matching [7], linguistic scene description [8], spatial databases [9]).

The histogram of forces lends itself, with great flexibility, to fuzzy definitions of directional spatial relations (such as "to the right of", "in front of", etc.) [8]. In this paper, we consider its use for the modeling of other important relationships: "surrounded by," "between" and "among." They are represented by fuzzy relations: s, b and a. Although the preposition "between" usually denotes a ternary relation, its model b is binary, as are s and a. For instance, the degree of truth of the proposition "A is between B_1 and B_2" is $b(A, B_1 \cup B_2)$, and the degree of truth of "A is among B_1, B_2, B_3 and B_4" is $a(A, \cup B_i)$. Contrary to "between," the term "surrounded by" does not belong to the list of primitive names for spatial relations drawn up by Freeman in [1]. However, it can be related to the element that Freeman refers to as "inside." The relation "surround" then corresponds to "outside." It is the semantic inverse of "surrounded by," *i.e.*, the propositions "A is surrounded by B_1, B_2 and B_3" and "B_1, B_2 and B_3 surround A" are equivalent. The term "among" does not belong to Freeman's list either. We call "among" a spatial relationship similar to "between." The main difference is that "between" usually involves three objects (A, and B_1 and B_2) whereas "among" involves four objects or more (A, and at least three other objects).

Fuzzy definitions of "surrounded by," "between" and "among" are proposed in Section III, and tested on real data in Section IV. Concluding remarks are given in Section V. Section II covers some existing models and briefly presents the notion of the histogram of forces.

II. BACKGROUND

A. The Histogram of Forces

The relative position of a 2D object A with regard to another object B can be represented by a function F^{AB} from $\mathbb{R}$ into $\mathbb{R}_+$ called the *histogram of forces associated with (A,B) via F*, or the *F-histogram associated with (A,B)*. For any direction θ, the value $F^{AB}(\theta)$ is the scalar resultant of elementary forces. These forces are exerted by the points of A on those of B, and each tends to move B in direction θ (Fig. 1). Actually, the letter F denotes a numerical function. Consider any real number r. If the elementary forces are in inverse ratio to d^r, where d represents the distance between the points considered, then F is denoted by F_r. The F_0-histogram (histogram of constant forces) and the F_2-histogram (histogram of gravitational forces) have very different and very interesting characteristics. The former provides a global view of the situation. It gives equal consideration to both the object's closest and farthest parts, whereas the F_2-histogram focuses on the closest parts.
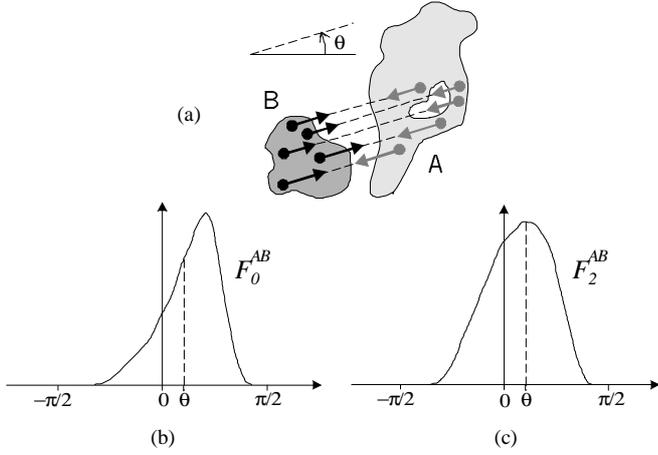
Fig. 1. Force histograms. (a) $F^{AB}(\theta)$ is the scalar resultant of forces (black arrows). Each one tends to move B in direction θ. (b) The histogram of constant forces associated with (A,B). It represents the position of A relative to B. (c) The histogram of gravitational forces associated with (A,B). It is another representation of the relative position between A and B.

The histogram of forces encapsulates structural information about the objects as well as information about their spatial relationships. It offers solid theoretical guarantees and nice geometric properties, ensures fast and efficient processing of vector data as well as of raster data, and enables the handling of fuzzy objects as well as crisp objects, intersecting objects as well as disjoint objects, and unbounded objects as well as bounded objects. Force histogram computation benefits from the power of integral calculus, is highly parallelizable, and utilizes a well-known algorithm that is commonly circuit coded in visualization systems. Details can be found in [4][5][7][8].

B. Existing Spatial Models

Spatial relationships like "surrounded by," "between" and "among" play an important role in the interpretation of a scene, and several methods of assessing these relationships have been proposed. There are two main approaches. The first one is based on such rules as (R_1) and (R_2) [10][11]:

(R_1) IF B_1 is to the *right* of A AND B_2 is to the *left* of A
     THEN A is *between* B_1 and B_2
(R_2) IF B is to the *right* of A AND B is *above* A AND
     B is to the *left* of A AND B is *below* A
     THEN A is *surrounded* by B

The main weakness of this approach comes from the fact that it is very difficult to express the *necessary* conditions for a spatial configuration to occur (instead of *sufficient* conditions). In the case of Fig. 2(a) for instance, the proposition "A is between B_1 and B_2" cannot be assessed using (R_1). Only a low (and useless) lower bound of its degree of truth would be obtained. Moreover, a rule like (R_2) assumes that an object can be in many directions with respect to another. This way of modeling directional relationships is questionable: generally, people do not combine more than two spatial preposi-

tions when translating visual information into natural language descriptions [12][13]. Lastly, one should be aware of the implicit assumptions on the objects. (R_2), for instance, should be applied only if B does not intersect the convex hull of A, *i.e.*, if it is known that A does not surround B at all. Indeed, the directional relationships are tied by the *semantic inverse* principle [1] (*e.g.*, A is to the left of B as much as B is to the right of A). Therefore, without constraints on the objects, there is no way to know which object surrounds (or includes!) the other. This is illustrated by Figs. 2(b)(c).

In the second approach, the considered relationships are not derived from other "even-more-primitive" relationships. Instead they are assessed directly. Most examples can be found in the modeling of "surrounded by" and are based on the computation of a histogram of angles. For any pixel P of A, let θ_P be the angle made by the two tangents from P to B as in Fig. 3. To each element θ of $[0,2\pi]$, the histogram associates the number of pixels P such that the angle θ_P is equal to θ. In [14], the degree of truth for "A is surrounded by B" is produced by a multilayer perceptron fed by the histogram values. Other authors resort to a decreasing membership function μ from $[0,2\pi]$ into $[0,1]$ such that $\mu(\theta)$ is 1 if θ is 0 and is 0 if θ is greater than π. In [11] for instance, the histogram of angles is assimilated to a fuzzy set and matched to μ. In [2], it is used to compute the aggregated value of the $\mu(\theta_P)$ when P describes A. The spatial models above all derive from Rosenfeld's visual surroundedness [15]. They assume that the object B is connected and does not intersect A. They are not extremely robust since the results are not sensitive to the thickness of the surrounding object, only to tangency points. Moreover, they cannot handle vector data, and are computationally expensive.
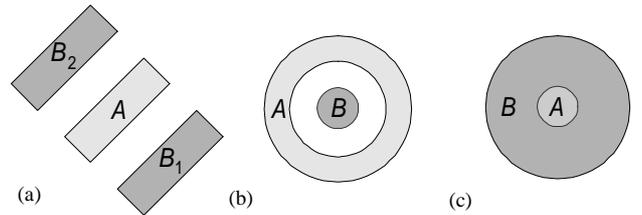

Fig. 2. (a) "B_1 is to the *right* of A" and "B_2 is to the *left* of A" are true to a certain extent only. Most fuzzy models [16] would produce degrees of truth lower than 0.5. However, A is clearly *between* B_1 and B_2. In (b) as in (c), the same fuzzy models would assess B to be somewhat *above*, *below*, to the *right* and to the *left* of A as well. It does not mean that A is *surrounded* by B. In (b), A surrounds B. In (c), A is *included* in B.
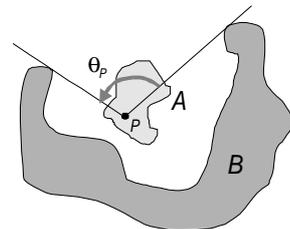

Fig. 3. Angle histogram computation for the modeling of "surrounded by."

III. NEW SPATIAL MODELS

In this section, two objects, A and B, are considered. The normalized histogram $F_r^{BA} / max_\theta F_r^{BA}(\theta)$ is denoted by H and assimilated to a fuzzy set (Fig. 4(a)). The symbol H^α denotes the support of H if α is 0, and the cut of level α if α belongs to the interval $(0,1]$ (Fig. 4(b)). For any α, the set H^α is represented by a polar diagram (Fig. 4(c)). Imagine that A and B are the objects of Fig. 4(d) (B is composed of four connected components). When the radius of A and the thickness of B both tend towards 0, the normalized histogram H becomes crisp, and the polar representation of any H^α becomes exactly like Fig. 4(c), *i.e.*, it merges with the configuration in the object space.

With any set H^α we associate $2q$ angles: z_1, z_2, …, z_q ("z" as in *z*ero) and o_1, o_2, …, o_q ("o" as in *o*ne). All these angles belong to the interval $[0,2\pi]$. Moreover: $z_1 \geq z_2 \geq ... \geq z_q$ and $o_1 \geq o_2 \geq ... \geq o_q$. The value q is the number of arcs in the polar representation of H^α. This is illustrated by Fig. 4(e). When H^α is equal to the referential set (*i.e.*, the set of real numbers), $q=1$ and $z_1=0$ and $o_1=2\pi$ (case of Fig. 5(a)). If q is greater than 1, two other angles y_1 and y_2 can be defined (Fig. 4(e)). We have: $y_1 \geq z_1 + z_2$ and $y_2 \geq z_1 + z_2$.

A. "Surrounded by" α-Cuts

The degree to which the set H^α describes a "surrounded" situation is denoted by $s^\alpha(A,B)$. It is a real number that belongs to the interval $[0,1]$. We propose the simple expression below, where k denotes some positive value (in our experiments, $k=1$). The highest possible degree, 1, is reached when z_1 is 0 (see Fig. 5(a)). The higher z_1, the lower $s^\alpha(A,B)$ (see Figs. 5(b)(d)). Note that $s^\alpha(A,B)$ does not depend on the number of arcs that are around the center point in the polar representation of H^α (as far as z_1 remains the same).

$$s^\alpha(A,B) = max\left(0, 1 - k\frac{z_1}{\pi}\right)$$

B. "Between" α-Cuts

The degree to which the set H^α describes a "between" situation is denoted by $b^\alpha(A,B)$. We consider that the relationship should not apply when q is 1, and that the highest values should be reached when q is 2 (*i.e.*, when exactly two arcs are around the center point in the polar representation of H^α). However, z_1 and z_2 should be close to each other, *i.e.*, the ratio z_2/z_1 should be high (Fig. 6(a)). There is nothing preventing z_1 and z_2 from being small (Fig. 6(b)), but $min(y_1, y_2)$ should not be too small (Fig. 6(c)). The situation is not as ideal when q is greater than 2. In accordance with Figs. 6(d)(e)(f), we consider that z_3 should be small compared to z_1 and z_2, *i.e.*, the higher z_3/z_2, the lower $b^\alpha(A,B)$. In conclusion, we propose the definition below, where k, k' and k'' denote three positive real numbers (in our experiments, $k=1$, $k'=1$ and $k''=2$).

$$q=1 \Rightarrow b^\alpha(A,B)=0$$

$$q=2 \Rightarrow b^\alpha(A,B)=min\left(1, k\frac{z_2}{z_1}, k'\frac{y_1}{\pi}, k'\frac{y_2}{\pi}\right)$$

$$q>2 \Rightarrow b^\alpha(A,B)=min\left(1, k\frac{z_2}{z_1}, k'\frac{y_1}{\pi}, k'\frac{y_2}{\pi}, max\left(0, 1-k''\frac{z_3}{z_2}\right)\right)$$

C. "Among" α-Cuts

The degree to which the set H^α describes an "among" situation is denoted by $a^\alpha(A,B)$. When q is lower than 3, we consider that the relationship should not apply. When q is greater than or equal to 3, we expect the arcs in the polar representation of H^α to be comparable in size and evenly distributed around the center point. The best case is illustrated by Figs. 7(a)(d), in which $z_1=z_q$ and $o_1=o_q$, *i.e.*, $z_1=(\Sigma_i z_i)/q$ and $o_1=(\Sigma_i o_i)/q$. The worst cases are illustrated by Figs. 7(b)(e), in which either $z_1 \gg z_2$ or $o_1 \gg o_2$, *i.e.*, $z_1 \approx \Sigma_i z_i$ or $o_1 \approx \Sigma_i o_i$. Hence the proposed definition below:
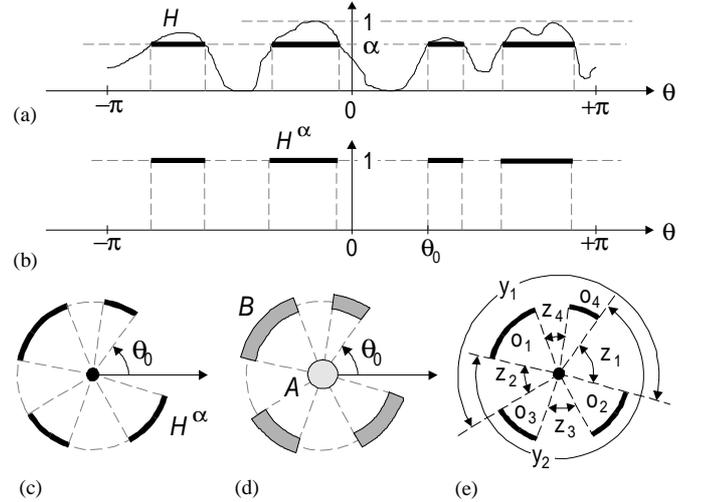

Fig. 4. (a) The normalized force histogram H is assimilated to a fuzzy set. (b) H^α is the cut of level α. (c) Polar representation of H^α. We would get a very similar diagram if the objects A and B were as in (d). (e) Angles associated with H^α.
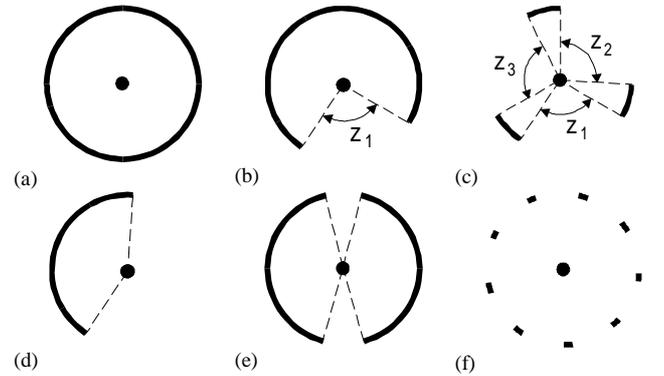

Fig. 5. Degree to which H^α describes a "surrounded" situation.
(a) $s^\alpha(A,B)=1$. (b) $s^\alpha(A,B)\approx0.5$. (c) $s^\alpha(A,B)\approx0.5$.
(d) $s^\alpha(A,B)=0$. (e) $s^\alpha(A,B)\approx0.9$. (f) $s^\alpha(A,B)\approx0.8$.

$$q<3 \Rightarrow a^{\alpha}(A,B) = 0$$

$$q\geq 3 \Rightarrow a^{\alpha}(A,B) = min\,(\,\frac{\Sigma_i z_i - z_1}{\Sigma_i z_i - (\Sigma_i z_i)/q}\,,\,\frac{\Sigma_i o_i - o_1}{\Sigma_i o_i - (\Sigma_i o_i)/q}\,)$$

D. Wrapping Computation Scheme

The spatial relationships "surrounded by," "between" and "among" are modeled by fuzzy binary relations: s, b and a. One might consider that the histogram values are not really useful, and that the most important is the knowledge of the directions in which forces appear—whatever the type and the amplitude of these forces. The relations s, b and a could, therefore, be defined as follows: $s(A,B)=s^0(A,B)$, $b(A,B)=b^0(A,B)$ and $a(A,B)=a^0(A,B)$. The method, however, is drastic. Moreover, it is not extremely robust. Although the values $s^{\alpha}(A,B)$, $b^{\alpha}(A,B)$ and $a^{\alpha}(A,B)$ vary in a continuous manner when an arc or a gap in the polar representation of H^{α} extends gradually at one of its ends, continuity is generally disrupted when an arc stretches to merge with its neighbor or shrinks to disappear.

All H^{α} sets should therefore be considered. In practice, the fuzzy set H can be represented by a finite number of α-cuts: H^{α_1}, H^{α_2}, …, H^{α_n}. The values α_1, α_2, …, α_n are such that: $\alpha_1=1>\alpha_2>…>\alpha_n>\alpha_{n+1}=0$ and $H^{\alpha_1}\subset H^{\alpha_2}\subset…\subset H^{\alpha_n}=H^0$.
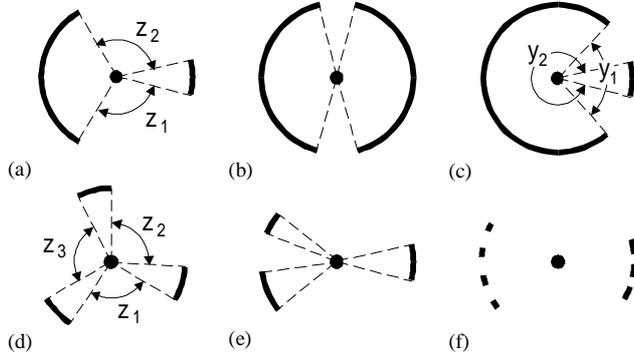

Fig. 6. Degree to which H^{α} describes a "between" situation.
(a) $b^{\alpha}(A,B)=1$. (b) $b^{\alpha}(A,B)=1$. (c) $b^{\alpha}(A,B)\approx0.4$.
(d) $b^{\alpha}(A,B)=0$. (e) $b^{\alpha}(A,B)\approx0.5$. (f) $b^{\alpha}(A,B)\approx0.8$.
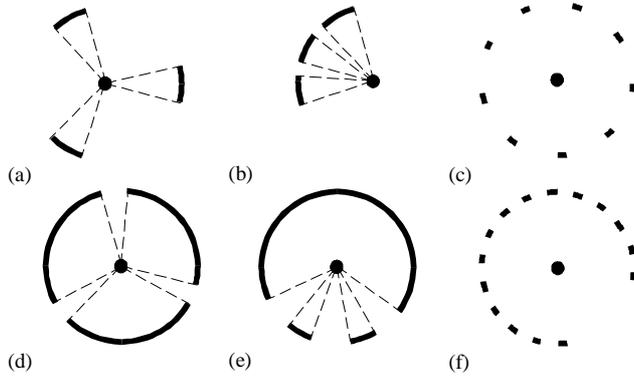
Fig. 7. Degree to which H^{α} describes an "among" situation.
(a) $a^{\alpha}(A,B)=1$. (b) $a^{\alpha}(A,B)\approx0.1$. (c) $a^{\alpha}(A,B)=1$.
(d) $a^{\alpha}(A,B)=1$. (e) $a^{\alpha}(A,B)\approx0.2$. (f) $a^{\alpha}(A,B)\approx0.8$.

The degrees of truth $s(A,B)$, $b(A,B)$ and $a(A,B)$ can then be defined as follows, using the general computation scheme proposed by Dubois and Jaulent in [17]:

$$s(A,B) = \Sigma_i\,m_i\,s^{\alpha_i}(A,B), \qquad (1)$$
$$b(A,B) = \Sigma_i\,m_i\,b^{\alpha_i}(A,B) \qquad (2)$$
$$\text{and} \quad a(A,B) = \Sigma_i\,m_i\,a^{\alpha_i}(A,B), \qquad (3)$$
$$\text{where} \quad m_i=\alpha_i-\alpha_{i+1}. \qquad (4)$$

The results presented in [17] rely on Shafer's theory of belief functions [18] and its links with fuzzy set and possibility theory [19]. Such links make statistical interpretation of membership functions possible [17]. For instance, the spatial relationship "between" measured on (A,B) yields the following probability distribution:

$$p_b\,|\,[0,1] \to [0,1]$$
$$x \mapsto \Sigma_i\,m_i\,\delta(x,b^{\alpha_i}(A,B))$$

where $\delta(x,b^{\alpha_i}(A,B))$ is 1 if x is equal to $b^{\alpha_i}(A,B)$ and is 0 otherwise. The degree $b(A,B)$ as stated in Equation (2) is simply the corresponding expected value. Instead of this single number, a fuzzy interval could also be obtained by transforming p_b into a possibility distribution [17]. In this paper, Equation (4) will be replaced by Equation (5) below, where f denotes a continuous non-increasing function from $[0,1]$ to $\mathbb{R}_+$. If $f(x)$ is 1, then (4) and (5) are equivalent. Four examples of functions f are given by Fig. 8.

$$m_i = \frac{\int_{\alpha_{i+1}}^{\alpha_i} f(x)dx}{\int_0^1 f(x)dx} \qquad (5)$$
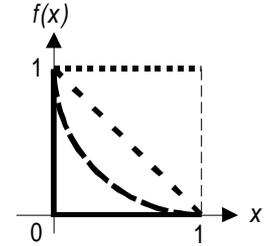

Fig. 8. Examples of functions f for the definition of basic probability assignments.

The L-shaped function corresponds to the case where the sets H^{α} are all ignored but H^0. As mentioned above, the support H^0 seems to be the most appropriate set for describing the relationships between the objects A and B (whereas, paradoxically, the core H^1 seems to be the less significant). Considering different types of force, and other basic probability assignments than the ones defined by (4), makes it possible to design spatial models that are more or less sensitive to the variability of the forces at work, *i.e.*, to the relative thickness of the objects and to the relative distance between them. This is illustrated by Fig. 9 and Table I. When relying on constant forces (F_0), the "between" models do not care about the increasing distance in situations *4*, *5* and *6*. However, they are very sensitive to the length of the right rectangle in situations *1*, *2* and *3*. This sensitivity can be controlled by f. With gravitational forces (F_2), it is just the opposite. The models pay special attention to relative distance and much less to
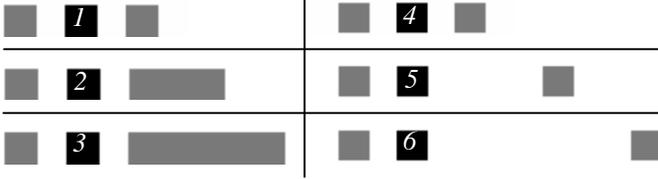
Fig. 9. Different "between" situations.
A is the black square, *B* is the union of the two gray rectangles.

TABLE I
THE VALUES OF $b(A,B)$ FOR THE SITUATIONS IN FIG. 9 DEPEND ON THE
FORCE HISTOGRAM AND ON THE BASIC PROBABILITY ASSIGNMENT

	F_0				F_2			
	∟	⌐	╲	—	∟	⌐	╲	—
1	1.00	1.00	1.00	1.00	1.00	*1.00*	1.00	1.00
2	1.00	0.77	0.58	0.35	1.00	*0.96*	0.87	0.64
3	1.00	0.57	0.38	0.21	1.00	*0.93*	0.81	0.56
4	1.00	1.00	1.00	1.00	1.00	*1.00*	1.00	1.00
5	1.00	1.00	1.00	1.00	1.00	*0.42*	0.26	0.14
6	1.00	1.00	1.00	1.00	1.00	*0.19*	0.10	0.05

relative thickness. For our experiments, in Section IV, we used F_2-histograms and the basic probability assignments defined by the arc-shaped function. They gave the best intuitive results (according to our own perception).

IV. EXPERIMENTS

We considered two LADAR (Laser Radar) images provided by the Naval Air Warfare Center (Fig. 10). Both were used in previous works to test different fuzzy systems for automatic target recognition [20] and linguistic scene description [8]. The system presented in [8] relies on the computation of force histograms. It outputs descriptions that make use of spatial prepositions related to directional relationships. For instance, the object *A* in Fig. 10(b) is found to be "perfectly to the right of B_1," "below-right of B_2," etc. The histograms computed are F^{AB_1}, F^{AB_2}, etc. Consider the group of buildings *4*, *5* and *6* in Fig. 10(d). Relative to that group, the tower *1* is "to the left but a little above," the tower *2* is "perfectly to the left," and the storehouse *3* is "perfectly above but slightly shifted to the left." Now, consider Fig. 10(b) again. If the system was asked to describe the position of *A* with respect to $\cup B_i$, it would have to generate $F^{(\cup B_i)A}$. This is an easy task, because $F^{(\cup B_i)A}=\Sigma_i \, F^{B_iA}$ and: $\forall \theta \in \mathbf{R}$, $F^{BA}(\theta)=F^{AB}(\theta-\pi)$ [7]. The first equation comes from the fact that forces are additive and $\cap B_i$ is empty. Knowing $F^{(\cup B_i)A}$, the system would then be able to output the requested linguistic description. However, in this particular case, it would come to the conclusion that none of the directional relationships are relevant, and would display the message "???????." We now have the ability to extend the system.

Fig. 11 shows ten configurations extracted from Figs. 10(b)(d). For each configuration, the degrees of truth $s(A,B)$, $b(A,B)$ and $a(A,B)$ have been computed (see Table II). The highest degree, if greater than some threshold (0.5), indicates
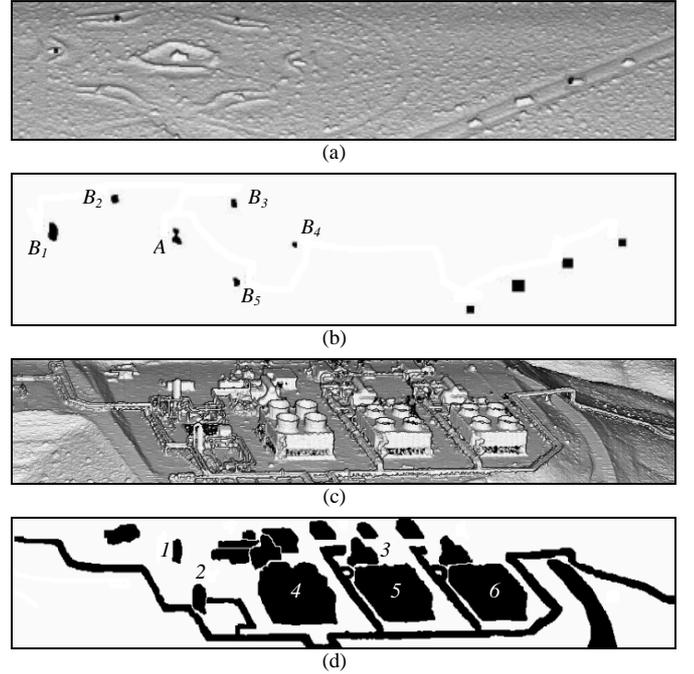
Fig. 10. (a) Filtered LADAR range image of a Surface-to-Air Missile site with convoy. (b) Objects detected by the Automatic Target Recognition system described in [20]. (c) Filtered LADAR range image of a power plant. (d) Hand-segmented image used in [8].

the spatial relationship which suits the configuration best. This relationship is also shown in Fig. 11. In Fig. 11(g), the fuzzy system presented in [8] detects a large amount of ambiguity. It states that "*A* is loosely below *B*" and notes that "the description is unsatisfactory." The second part of the message prompts the user to turn to relationships other than directional ones. The extended system would stick to this output anyhow, because the values $s(A,B)$, $b(A,B)$ and $a(A,B)$ are too low. In Fig. 11(j), the amount of ambiguity detected by the original system is even larger, and no appropriate description can be given. Although the value $s(A,B)$ is quite high, one should not state that "*A* is surrounded by *B*." As happens with the rule-based methods (Section II.*B*), it is important to make sure beforehand that *B* does not intersect the convex hull of *A* (or that *A* is "rather" compact, and that *A* and *B* do not intersect "much")—which is not the case here. This is the price to pay for concurrent assessment of directional relationships and spatial relationships like "surrounded by," "between" and "among." Finally, here are some results obtained when choosing other basic probability assignments (see Section III.*D*). With the standard assignments (Equation (4)), the highest degrees of truth are often lower than 0.3, and no relationship really stands out. When f is the steep L-shaped function (the α-cuts of the fuzzy set H are ignored, only the support H^0 is considered), the results tend to be crisp, and are much less satisfactory (according to our own perception). In Fig. 11(d) for instance, *A* is found to be "between *B*" ($b(A,B)=0.85$ and $a(A,B)=0.72$); for Fig. 11(g), we get $s(A,B)=0.91$ (instead of 0.41); in Fig. 11(h), *A* is "surrounded by *B*" ($s(A,B)=0.94$ and $b(A,B)=0.00$); in Fig. 11(i), we have $s(A,B)=1.00$ (instead of 0.63).
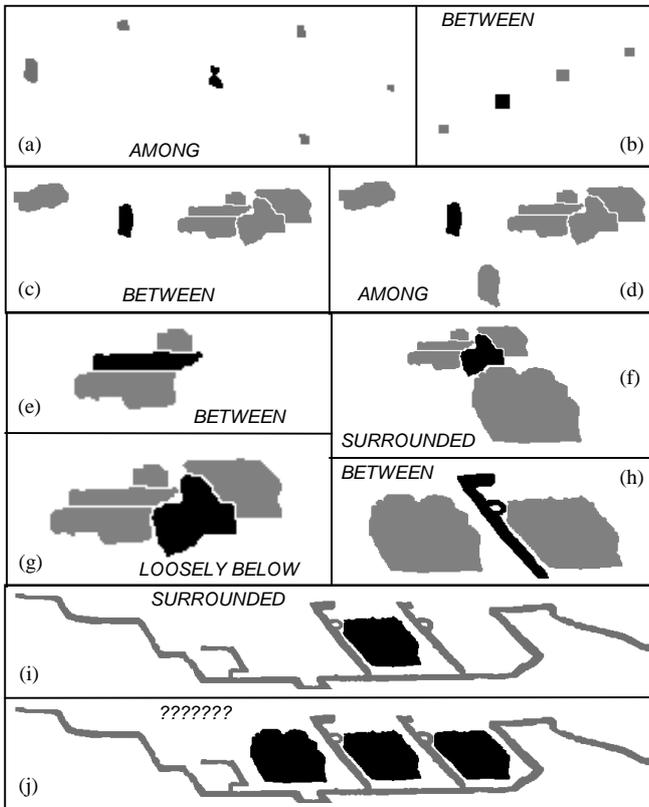
Fig. 11. Ten configurations extracted from the images shown in Fig. 10.. *A* is the black object, *B* is the union of the gray ones.

TABLE II

VALUES OF $s(A,B)$, $b(A,B)$ AND $a(A,B)$
FOR THE CONFIGURATIONS DEPICTED IN FIG. 11

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
s	0.23	0.08	0.07	0.23	0.58	*0.92*	0.41	0.58	*0.63*	0.68
b	0.29	*0.92*	*0.71*	0.47	*0.62*	0.02	0.01	*0.61*	0.16	0.12
a	*0.71*	0.00	0.00	*0.67*	0.06	0.02	0.01	0.00	0.00	0.00

V. CONCLUSION

We have shown that the histogram of forces can be employed to design consistent fuzzy models of spatial relationships like "surrounded by," "between" and "among." Compared to other existing methods, force histogram-based methods are computationally much less expensive; they are able to handle non-connected objects and groups of objects; they are able to handle vector data as well as raster data; and a variety of spatial models, more or less sensitive to the thickness of the objects and to the distance between them, can be defined. Another important advantage is that the directional relations (*i.e.*, "to the left of," "above," etc.) can be assessed concurrently. Unfortunately, there is a price to pay: as happens with the rule-based methods, we have to make sure beforehand that the considered objects satisfy certain constraints (*e.g.*, one object is compact and does not intersect the other object, or one of the objects does not intersect the convex hull of the other object). Only specific tools, dedicated to the evaluation of the spatial relationships studied here, can overcome this limitation. Designing new types of force histograms constitutes a promising avenue. The idea is to adopt novel sets of axiomatic properties, and to change the way the longitudinal sections are handled [4][5].

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