Combined Extraction of Directional and Topological Relationship Information from 2D Concave Objects

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Abstract. *The importance of topological and directional relationships between spatial objects has been stressed in different fields, notably in Geographic Information Systems (GIS). In an earlier work, we introduced the notion of the F-histogram, a generic quantitative representation of the relative position between two 2D objects, and showed that it can be of great use in understanding the spatial organization of regions in images. Here, we illustrate that the F-histogram constitutes a valuable tool for extracting directional and topological relationship information. The considered objects are not necessarily convex and their geometry is not approximated through, e.g., Minimum Bounding Rectangles (MBRs). The F-histograms introduced in this chapter are coupled with Allen's temporal relationships based on fuzzy set theory. Allen's relationships are commonly extended into the spatial domain for GIS purposes, and fuzzy set theoretic approaches are widely used to handle imprecision and achieve robustness in spatial analysis. For any direction in the plane, the F-histograms define a fuzzy 13-partition of the set of all object pairs, and each class of the partition corresponds to an Allen relation. Lots of directional and topological relationship information as well as different levels of refinements can be easily obtained from this approach, in a computationally tractable way.*

Keywords. *F-histograms, Allen relations, spatial relations, spatial analysis, Geographic Information Systems, fuzzy sets.*

1 Introduction

Space plays a fundamental role in human cognition. In everyday situations, it is often viewed as a construct induced by spatial relations, rather than as a container that exists independently of the objects located in it. A variety of formalisms developed in Artificial Intelligence naturally deal with space on the basis of relations between objects. Geographic Information Systems constitute a wide area of applications for such formalisms. Many authors, from different fields, have stressed the importance of topological (Allen 1983; Clementini and Di Felice

1997; Cohn et al. 1997; Kuipers 1978) and directional relationships (Bloch 1999; Dutta 1991; Krishnapuram et al. 1993; Kuipers and Levitt 1988). Work in the modeling of these relationships for GIS is often based on an extension into the spatial domain of Allen's temporal relationships (Allen 1983). A common procedure is to approximate the geometry of spatial objects by Minimum Bounding Rectangles (Clementini et al. 1994; Nabil et al. 1995; Sharma and Flewelling 1995). A 2D object is then represented as a set of two perpendicular 1D segments and relationships between objects are inferred from relationships between segments. To enhance querying and improve accuracy in relationship determination, however, several alternatives and refinements have been proposed. In (Petry et al. 2002), for instance, MBRs are partitioned into sets of rectangles. Such partitioning results in a finer approximation of the object's true geometry, called Multiple Rectangle Representation.

The need to handle imprecise and uncertain information concerning spatial data has been widely recognized in recent years, e.g., (Goodchild and Gopal 1990), and there has been a strong demand in the field of GIS for providing approaches that deal with such information. Humans often deal with space on a qualitative basis, allowing for imprecision in spatial descriptions when interacting with each other. Qualitative spatial reasoning, a subfield of AI, aims at modeling commonsense knowledge of space (Cohn 1995). Computational approaches for spatial modeling and reasoning, however, can benefit from more quantitative measures. For instance, qualitative composition of positional relations, if iterated over a path of several intermediate positions, introduces too much indeterminacy in the result. The problem can be addressed by coupling qualitative with fuzzy, semi-quantitative knowledge (Clementini 2002). As many authors early emphasized, fuzzy approaches are of great interest for spatial modeling and reasoning (Dutta 1991; Freeman 1975; Robinson 1988; Wang et al. 1990). Research on fuzzy sets and GIS is very active. A recent special issue of *Fuzzy Sets and Systems* (Cobb et al. 2000), for instance, touches on topics as varied as fuzzy objects for GIS, fuzzy spatial queries and landform classification with fuzzy k-means.

In earlier publications, we introduced the notion of the F-histogram (Matsakis 1998; Matsakis and Wendling 1999). It is a generic quantitative representation of the relative position between two 2D objects. It encapsulates structural information about the objects as well as information about their spatial relationships. It is sensitive to the shape of the objects, their orientation and their size. It is also sensitive to the distance between them. Moreover, the F-histogram enables the handling of intersecting, concave, non-connected, unbounded, fuzzy objects as well as of disjoint, convex, bounded, crisp objects. Most work focused on particular F-histograms called force histograms. These histograms offer solid theoretical guarantees and nice geometric properties (Matsakis et al., to appear). They ensure fast and efficient processing of vector data (Skubic et al. 2003) as well as of raster data (Matsakis et al. 2001). Numerous applications have been studied, and new applications continue to be explored. For instance, the histogram of forces lends itself, with great flexibility, to the definition of fuzzy spatial relations. The fuzzy directional relations described in (Matsakis et al. 2001) preserve important relative position properties and can provide inputs to systems for linguistic scene description. One such system has been developed and dedicated to human-robot communication (Skubic et al. 2003). Reference (Matsakis 2002) reviews and classifies work on the histogram of forces. It shows that the notion of the F-histogram can be of great use in understanding the spatial organization of regions in images.

The aim of this chapter is to illustrate that the F-histogram, because of its general properties, constitutes a valuable tool for extracting directional and topological relationship information from two objects. The objects considered here are 2D, crisp, bounded objects, but they are not necessarily convex, nor connected, and they may have holes in them. Their geometry is not approximated through, e.g., centroids, MBRs or convex hulls. The F-histograms described in the present work are coupled with Allen relations using fuzzy set theory. Obviously, the set of Allen relations does not allow all possible topological relationships between 2D concave objects to be described. However, it is a well-known set, of reasonable size, which has been extensively used. For any oriented line, Δ, the Allen relations define a crisp 13-partition of the set of pairs of segments on Δ. For any direction, θ, the F-histograms introduced here define a fuzzy 13-partition of the set of all object pairs, and each class of the partition corresponds to an Allen relation. Lots of directional and topological relationship information as well as different levels of refinements can be easily obtained from this approach, in a computationally tractable way. The notion of the F-histogram is described in Section 2 and the way F-histograms are coupled with Allen relations is examined in Section 3. Preliminary experiments validate the approach in Section 4 and conclusion is given in Section 5.

2 When Pairs of 2D Objects Are Handled as Pairs of 1D Sections

We describe here the notion of the F-histogram (Section 2.2), which was introduced in an earlier work (Matsakis 1998; Matsakis and Wendling 1999). F-histograms include f-histograms (Section 2.3) and f-histograms include φ-histograms (Section 2.4). Most of the previous research has focused on force histograms, which are particular φ-histograms and have shown to be of great use in understanding the spatial organization of image objects (Section 2.5). First of all, we go over some terms and introduce a few notations (Section 2.1).

2.1 Terminology and Notations

As shown in Fig. 1, the plane reference frame is a positively oriented orthonormal frame (O, $\vec{i}$, $\vec{j}$). For any real numbers α and v, the vectors $\vec{i}_\alpha$ and $\vec{j}_\alpha$ are the respective images of $\vec{i}$ and $\vec{j}$ through the α-angle rotation, and $\Delta_\alpha(v)$ is the oriented line whose reference frame is defined by $\vec{i}_\alpha$ and the point of coordinates $(0,v)$—relative to $(O,\vec{i}_\alpha,\vec{j}_\alpha)$. The term *object* denotes a nonempty bounded set of points, E, equal to its interior closure[1], and such that for any α and v the intersection $E \cap \Delta_\alpha(v)$ is the union of a finite number of mutually disjoint segments. Note that an object may have holes in it and may consist of many connected components. The intersection $E \cap \Delta_\alpha(v)$, denoted by $E_\alpha(v)$, is a *longitudinal section* of E. Finally, the symbol T denotes the set of all triples $(\alpha, E_\alpha(v), G_\alpha(v))$, where α and v are any real numbers and E and G are any objects.

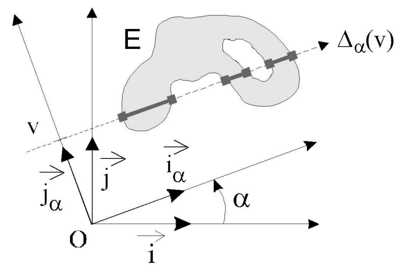

Fig. 1. Oriented straight lines and longitudinal sections. Here, $E_\alpha(v)=E \cap \Delta_\alpha(v)$ is the union of three disjoint segments.

2.2 F–Histograms

Consider two objects A and B (the *argument* and the *referent*), a direction θ and some proposition $\mathscr{P}^{AB}(\theta)$ like “A is *after* B in direction θ,” “A *overlaps* B in direction θ,” or “A *surrounds* B in direction θ.” We want to attach a weight to $\mathscr{P}^{AB}(\theta)$. To do so, the objects A and B are handled as longitudinal sections (Fig. 2).

- For each v, the pair $(A_\theta(v),B_\theta(v))$ of longitudinal sections is viewed as an argument put forward to support $\mathscr{P}^{AB}(\theta)$.
- A function F from T into $\mathbb{R}_+$ (the set of non-negative real numbers) attaches the weight $F(\theta,A_\theta(v),B_\theta(v))$ to this argument $(A_\theta(v),B_\theta(v))$.
- The total weight $F^{AB}(\theta)$ of the arguments stated in favor of $\mathscr{P}^{AB}(\theta)$ is naturally set to:

$$F^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta,A_\theta(v),B_\theta(v))\, dv.$$

- If the domain of the function F^{AB} so defined is all of $\mathbb{R}$ (the set of real numbers), then F^{AB} is called the F–*histogram associated with* (A,B). This histogram, which is a periodic function with period 2π, is one possible representation of the position of A with regard to B.

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[1] In other words, it is a 2D object that does not include any “grafting,” such as an arc or isolated point.

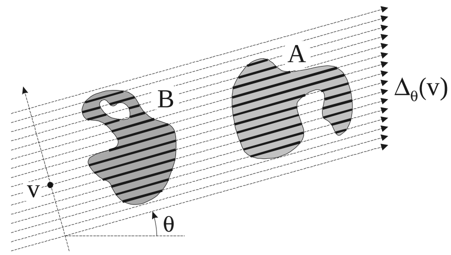


Fig. 2. The objects are handled as longitudinal sections:
$F^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A \cap \Delta_\theta(v), B \cap \Delta_\theta(v))\, dv$.

2.3 f–Histograms

There exists one set $\{I_i\}_{i \in 1..n}$ of mutually disjoint segments (and only one) such that $A_\theta(v) = \cup_{i \in 1..n} I_i$. Likewise, there exists one set $\{J_j\}_{j \in 1..m}$ of segments such that $B_\theta(v) = \cup_{j \in 1..m} J_j$. The function F, in charge of the longitudinal sections, might delegate the handling of these segments to some function f, from $\mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+$ into $\mathbb{R}_+$ (Fig. 3). The case is described below.

- Each (I_i,J_j) is considered an argument put forward to support the proposition $\mathcal{P}^{AB}(\theta)$.
- The function f attaches the weight $f(x_{I_i}, y^{\theta}_{I_i J_j}, x_{J_j})$ to this argument (I_i,J_j)—where x_{I_i} and x_{J_j} denote the lengths of I_i and J_j, and where $y^{\theta}_{I_i J_j}$ characterizes the relative position of I_i and J_j on $\Delta_\theta(v)$ (Fig. 4).
- $F(\theta,A_\theta(v),B_\theta(v))$ is naturally set to the sum of the weights $f(x_{I_i}, y^{\theta}_{I_i J_j}, x_{J_j})$ of all the (I_i,J_j) arguments: $F(\theta,A_\theta(v),B_\theta(v)) = \Sigma_{i \in 1..n,\, j \in 1..m}\ f(x_{I_i}, y^{\theta}_{I_i J_j}, x_{J_j})$.
- F^{AB} can then be renamed f^{AB} and called the f–*histogram associated with* (A,B).

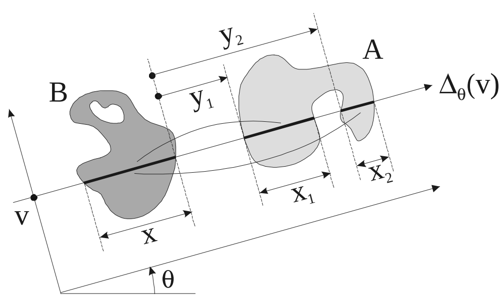


Fig. 3. The function F, in charge of the longitudinal sections, might delegate the handling of segments to some function f: $F(\theta,A \cap \Delta_\theta(v),B \cap \Delta_\theta(v)) = f(x_1,y_1,x) + f(x_2,y_2,x)$.

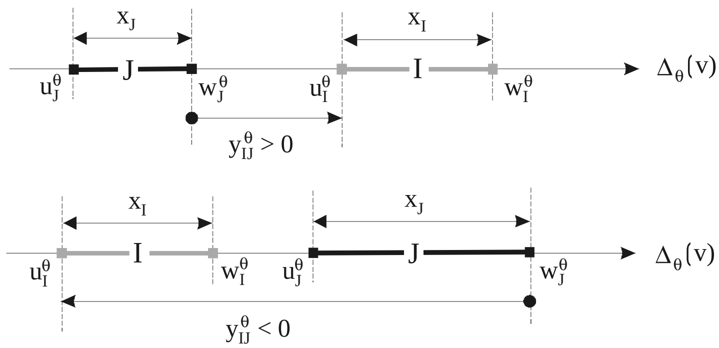


Fig. 4. A pair (I,J) of segments on an oriented line $\Delta_\theta(v)$ and the values attached to it.

2.4 φ–Histograms

In turn, f, which is in charge of the pairs (I,J) of segments, might delegate the handling of points to another function φ, from $\mathbb{R}$ into $\mathbb{R}_+$ (Fig. 5). The case is described below.

- Each (M,N), with M in I and N in J, is considered an argument put forward to support the proposition $\mathcal{P}^{AB}(\theta)$.
- The function φ attaches the weight φ(u−w) to this argument (M,N)—where u and w specify the location of M and N on $\Delta_\theta(v)$ and u−w characterizes the relative position of these points on $\Delta_\theta(v)$ (Fig. 5).
- $f(x_I, y_{IJ}^\theta, x_J)$ is naturally set to the sum of the weights φ(u−w) of all the (M,N) arguments:

  $$f(x_I, y_{IJ}^\theta, x_J) = \int_{y_{IJ}^\theta + x_J}^{x_I + y_{IJ}^\theta + x_J} \left(\int_0^{x_J} \varphi(u-w)\, dw \right) du.$$

  Note that:

  $$\int_{y_{IJ}^\theta + x_J}^{x_I + y_{IJ}^\theta + x_J} \left(\int_0^{x_J} \varphi(u-w)dw \right) du = \int_{u_I^\theta}^{w_I^\theta} \left(\int_{u_J^\theta}^{w_J^\theta} \varphi(u-w)dw \right) du$$
  $$= \int_{u_J^\theta}^{w_J^\theta} \left(\int_{u_I^\theta}^{w_I^\theta} \varphi(u-w)du \right) dw,$$

  where u_I^θ, w_I^θ, u_J^θ and w_J^θ represent the coordinates of the ends of the two segments I and J (Fig. 4).
- f^{AB} (or F^{AB}) can then be renamed φ^{AB} and called the φ–*histogram associated with* (A,B).

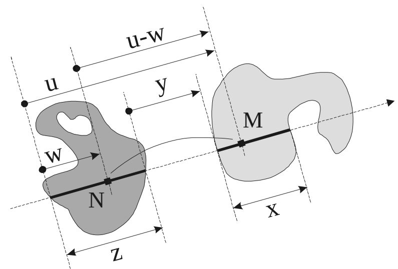


Fig. 5. The function f, in charge of the segments, might delegate the handling of points like M and N to some function φ: $f(x,y,z) = \int_{y+z}^{x+y+z} \left(\int_0^z \varphi(u-w)\, dw \right) du$.

2.5 Force Histograms vs. Other F-Histograms

In most previous work, the considered proposition $\mathcal{P}^{AB}(\theta)$ is "A is in direction θ of B" (i.e., "A is *after* B in direction θ") and the F-histograms are φ_r-histograms, where r is a real number and φ_r is the function from $\mathbb{R}$ into $\mathbb{R}_+$ defined by:

$$\forall d \in \mathbb{R},\ d \le 0 \Rightarrow \varphi_r(d)=0 \quad \text{and} \quad d>0 \Rightarrow \varphi_r(d)=1/d^r.$$

The value $\varphi_r^{AB}(\theta)$ can be seen as the scalar resultant of elementary forces. These forces are exerted by the points of A on those of B, and each tends to move B in direction θ (Fig. 6). The mapping φ_r defines the force fields. As an example, gravitational force fields can be represented by φ_2. This is according to Newton's law of gravity, which states that every particle attracts every other particle with a force inversely proportional to the square of the distance (i.e., d) between them. The argument A and the referent B can then be seen as flat metal plates of uniform density and constant and negligible thickness. A φ_r-histogram is called a *histogram of forces*. It offers solid theoretical guarantees and nice geometric properties. Numerous applications have been studied, and new applications continue to be explored. Reference (Matsakis 2002) reviews and classifies work on the histogram

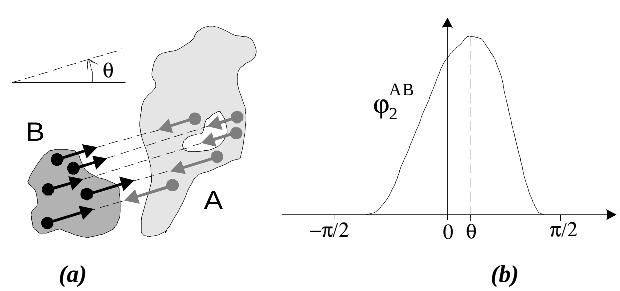


Fig. 6. Force histograms. *(a)* $\varphi_r^{AB}(\theta)$ is the scalar resultant of elementary forces (black arrows). Each one tends to move B in direction θ. *(b)* The histogram of gravitational forces associated with (A,B) is one possible representation of the position of A relative to B.

of forces. It touches on varied topics, such as the modeling of spatial relations, spatial indexing mechanisms for medical image databases, pattern recognition, scene matching, linguistic scene description and human-robot communication.

As said above, most work on F-histograms has focused on force histograms. The use of f-histograms that are not φ-histograms, however, was suggested in (Matsakis 1998) for the handling of convex objects. The use of F-histograms that are not f-histograms was suggested in (Matsakis and Andréfouët 2002) with the aim of attaching a weight to the proposition $\mathcal{P}^{AB}(\theta) \equiv$ "A *surrounds* B in direction θ." Malki et al. (2002) consider the propositions $\mathcal{P}_r^{AB}(\theta) \equiv$ "A *r* B in direction θ," where *r* belongs to the set {>, *mi*, *oi*, *f*, *d*, *si*, =, *s*, *di*, *fi*, *o*, *m*, <} of Allen relations (Fig. 7). For instance, $\mathcal{P}_>^{AB}(\theta)$ is "A is *after* B in direction θ" and $\mathcal{P}_o^{AB}(\theta)$ is "A *overlaps* B in direction θ." To attach a weight to these propositions, the authors rely on the research presented in (Matsakis 1998) and propose the use of f-histograms. The thirteen f-histograms are defined by the following functions:

- if y>0 then $f_>(x,y,z)=y/(x+y+z)$
  else $f_>(x,y,z)=0$
- if y=0 then $f_{mi}(x,y,z)=1$
  else $f_{mi}(x,y,z)=0$
- if (y<0 and x+y>0 and y+z>0) then $f_{oi}(x,y,z)=-y(1/x+1/z)$
  else $f_{oi}(x,y,z)=0$
- if (y<0 and x+y>0 and y+z=0) then $f_{si}(x,y,z)=z/x$
  else $f_{si}(x,y,z)=0$
- if (y<0 and x+y>0 and y+z<0) then $f_{di}(x,y,z)=z/x$
  else $f_{di}(x,y,z)=0$
- if (y<0 and x+y=0 and y+z>0) then $f_f(x,y,z)=x/z$
  else $f_f(x,y,z)=0$
- if (y<0 and x+y=0 and y+z=0) then $f_=(x,y,z)=x$
  else $f_=(x,y,z)=0$
- if (y<0 and x+y=0 and y+z<0) then $f_{fi}(x,y,z)=z/x$
  else $f_{fi}(x,y,z)=0$
- if (y<0 and x+y<0 and y+z>0) then $f_d(x,y,z)=x/z$
  else $f_d(x,y,z)=0$
- if (y<0 and x+y<0 and y+z=0) then $f_s(x,y,z)=x/z$
  else $f_s(x,y,z)=0$
- if (y<0 and x+y<0 and y+z<0 and x+y+z>0) then $f_o(x,y,z)=(x+y+z)(1/x+1/z)$
  else $f_o(x,y,z)=0$
- if (y<0 and x+y<0 and y+z<0 and x+y+z=0) then $f_m(x,y,z)=1$
  else $f_m(x,y,z)=0$
- if (y<0 and x+y<0 and y+z<0 and x+y+z<0) then $f_<(x,y,z)=y/(x+y+z)$
  else $f_<(x,y,z)=0$

Only convex objects are actually considered. Moreover, there is no real consistency between the f_r functions and, hence, between the f_r–histograms. For instance, the function $f_>$, which is continuous on its domain and whose range is [0,1], defines a fuzzy relation between aligned segments. The function f_{si} also defines a fuzzy relation between aligned segments; it is not, however, continuous on its domain; its range is [0,1). The function f_{mi} defines a crisp relation; its range is {0,1}. The function f_{oi} defines neither a crisp nor a fuzzy relation; its range is [0,2). In this chapter, we revisit the work of Malki et al. Note that, in their publications, the authors refer to the set of thirteen f_r–histograms as the *histogram of spatial relations*. They also use the term of *orientation histogram* instead of φ-histogram. We do not subscribe to these changes in terminology.

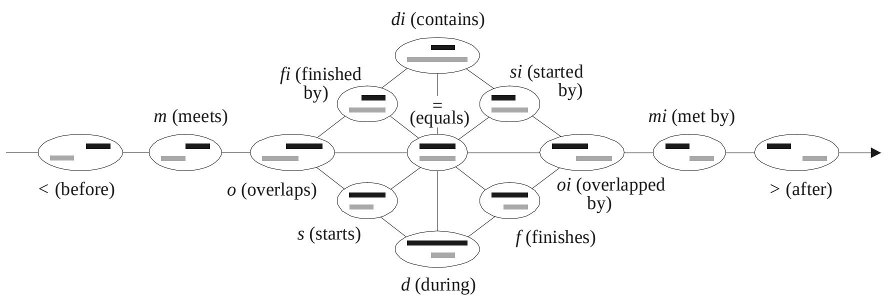


Fig. 7. Allen relations (Allen 1983) between two segments on an oriented line. The black segment is the referent, the gray segment is the argument. Two relations r_1 and r_2 are linked if and only if they are conceptual neighbors (Freksa 1992), i.e., r_1 can be obtained directly from r_2 by moving or deforming the segments in a continuous way.

3 When F-Histograms Are Coupled With Allen Relations Using Fuzzy Set Theory

Consider an Allen relation r, two objects A and B (convex or not) and a direction θ. The goal of this chapter is to attach an appropriate weight to the proposition $\mathcal{P}_r^{AB}(\theta) \equiv$ "A r B in direction θ" (see Section 2.5). As discussed in Section 2.2, each pair $(A_\theta(v),B_\theta(v))$ of longitudinal sections will be viewed as an argument put forward to support $\mathcal{P}_r^{AB}(\theta)$. A function F_r will attach the weight $F_r(\theta,A_\theta(v),B_\theta(v))$ to this argument and the total weight $F_r^{AB}(\theta)$ of the arguments stated in favor of $\mathcal{P}_r^{AB}(\theta)$ will be set to:

$$F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta,A_\theta(v),B_\theta(v))\, dv.$$

The question, therefore, is how to define F_r. Let us describe a very simple idea. Consider two segments I and J on an oriented line. We have either IrJ or ¬(IrJ). The first case can be rewritten r(I,J)=1 and the second case r(I,J)=0. Now, assume the oriented line is $\Delta_\theta(v)$ and I and J are the longitudinal sections $A_\theta(v)$ and $B_\theta(v)$. There exists one set $\{I_i\}_{i\in 1..m}$ of mutually disjoint segments such that: $I = \cup_{i\in 1..m} I_i$. Likewise, there exists one set $\{J_j\}_{j\in 1..n}$ of segments such that: $J = \cup_{j\in 1..n} J_j$. We could extend the thirteen Allen relations between segments to relations between longitudinal sections and say that r(I,J)=1 (i.e., $F_r(\theta,A_\theta(v),B_\theta(v)=1)$ if and only if there exist two segments I_i and J_j such that $r(I_i,J_j)=1$ (and r(I,J)=0 otherwise). The idea, obviously, is not very satisfactory. For instance, as shown by Figs. 8 to 10, small changes in the longitudinal sections could affect their relationships significantly. As mentioned in Section 1, fuzzy set theoretic approaches have been widely used to handle imprecision and achieve robustness in spatial analysis. The issue raised by Fig. 8 is addressed in Section 3.1 by fuzzifying the thirteen Allen relations. The issue raised by Fig. 9 is addressed in Section 3.2 by fuzzifying the longitudinal sections. Section 3.3 addresses the last issue (Fig. 10) and defines the function F_r.

Fig. 8. A single pixel at the end of one segment might change the relationships significantly. We may have (>(I,J)=1 and *mi*(I,J)=0 and *oi*(I,J)=0) or (>(I,J)=0 and *mi*(I,J)=1 and *oi* (I,J)=0) or (>(I,J)=0 and *mi*(I,J)=0 and *oi*(I,J)=1).

Fig. 9. A missing pixel in the middle of one segment might change the relationships significantly. *(a)* *mi*(I,J)=0 and *oi*(I,J)=1 and *d*(I,J)=0. *(b)* *mi*(I,J)=1 and *oi*(I,J)=0 and *d*(I,J)=1.

Fig. 10. A single pixel lost in the middle of nowhere might change the relationships significantly. *(a)* >(I,J)=1 and <(I,J)=0. *(b)* >(I,J)=1 and <(I,J)=1.

3.1 Fuzzification of Allen Relations

An Allen relation r can be fuzzified in many ways, depending on the intent of the work. Guesgen (2002), for instance, proceeds in a qualitative manner. Let (I,J) be a pair of segments and let r' be the only (crisp) Allen relation such that Ir'J. Denote by r(I,J) the degree to which the statement IrJ is to be considered true. r(I,J) is chosen as a decreasing function of the conceptional distance between r and r' (i.e., of the distance between r and r' on the graph shown in Fig. 7). Only a few membership values—which are to be provided by the user—can therefore be taken. Here, we proceed in a quantitative manner. Let α, β, γ and δ be four real numbers such that $\alpha<\beta\leq\gamma<\delta$ and let $\mu_{(\alpha,\beta,\gamma,\delta)}$ be the trapezoid membership function defined on the set of real numbers by:

$$\mu_{(\alpha,\beta,\gamma,\delta)}(u) = \max(\min(\frac{u-\alpha}{\beta-\alpha}, 1, \frac{\delta-u}{\delta-\gamma}), 0)$$

The support of the corresponding fuzzy set is the open interval (α,δ) and the core is $[\beta,\gamma]$: $\mu_{(\alpha,\beta,\gamma,\delta)}(u) \neq 0 \Leftrightarrow u \in (\alpha,\delta)$ and $\mu_{(\alpha,\beta,\gamma,\delta)}(u) = 1 \Leftrightarrow u \in [\beta,\gamma]$. The thirteen Allen relations are fuzzified as shown in Fig. 11. Each relation, except =, is defined by the min of a few trapezoid membership functions. For instance, the fuzzy relation *mi* associates with each pair (I,J) of segments the value

$$mi(I,J) = \mu_{(-a/2,0,0,a/2)}(y)$$

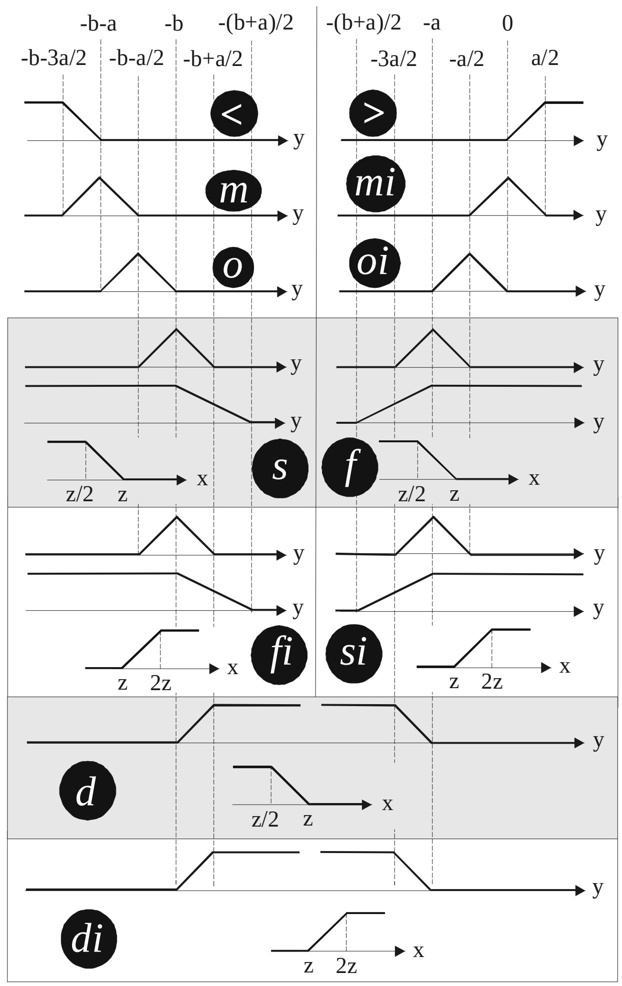
and the relation *f* associates with each (I,J) the value

$$f(I,J) = \min(\mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-\infty,z/2,z/2,z)}(x)).$$

Notations are as described in the caption of Fig. 11. Let $\mathcal{A}$ be the set of all thirteen fuzzy relations. Three properties are worth noticing. First, for any pair (I,J), we have: $\Sigma_{r\in\mathcal{A}}\, r(I,J) = 1$. This, of course, comes from the definition of = (and it can be shown that = takes its values in [0,1]). Second, for any r in $\mathcal{A}$, there exist pairs (I,J) such that $r(I,J)=1$. Lastly, for any pair (I,J) and any r_1 and r_2 in $\mathcal{A}$, if $r_1(I,J)\neq 0$ and $r_2(I,J)\neq 0$ then r_1 and r_2 are direct neighbors in the graph of Fig. 7.

3.2 Fuzzification of Longitudinal Sections

In this section, we address the issue raised by Fig. 9. The idea is to consider that if two segments are close enough relative to their lengths, then they should be seen, to a certain extent, as a single segment. Let I be the longitudinal section $E\cap\Delta_\theta(v)$ of some object E. Assume I is not empty. There exists one set $\{I_i\}_{i\in 1..n}$ of mutually disjoint segments (and only one) such that: $I = \cup_{i\in 1..n} I_i$. The indexing can be chosen such that, for any i in 1..n−1, the segment I_{i+1} is after I_i in direction θ. Let J_i be the open interval $H(I_i\cup I_{i+1})-I_i\cup I_{i+1}$, where $H(I_i\cup I_{i+1})$ denotes the convex hull of $I_i\cup I_{i+1}$, i.e., the smallest segment that contains both I_i and I_{i+1}. The longitudinal section I is considered a fuzzy set on $\Delta_\theta(v)$. Its membership function is μ_I and its α-cut is αI. For any point M on any I_i, the value $\mu_I(M)$ is 1. For any point M on any J_i, the value $\mu_I(M)$ is α_i —and, initially, $\alpha_i = 0$. The algorithm presented in Fig. 12 fuzzifies I by increasing these membership degrees α_i. An illustration of the fuzzification process is presented in Fig. 13. Note that the maximum number of iterations of the **`while`** loop is n.



$$= \;=\; 1 - \left(\begin{matrix} < + m + o + s + f + d + \\ > + mi + oi + si + fi + di \end{matrix} \right)$$

Fig. 11. The thirteen fuzzified Allen relations between two segments I and J on an oriented line. Each relation, except =, is defined by the min of a few membership functions (one for <, >, *m*, *mi*, *o*, *oi*; three for *s*, *si*, *f*, *fi*, *d* and *di*). x is the length of I (the argument), z is the length of J (the referent), a is min(x,z), b is max(x,z) and y characterizes the position of I relative to J (see Fig. 4).

```
c ← 0;
α ← 1;
while α > 0 do

   %-------------- There exists one set {I^c_i}i∈1..nc of
   mutually disjoint segments (and only one) such that:
   αI = ∪i∈1..nc I^c_i . For any i and any j in 1..nc, with i≠j,
   the length of I^c_i is denoted by x^c_i and the distance
   between I^c_i and I^c_j is denoted by d^c_ij. ----------------%

   for any i in 1..nc–1 do
        for any j in i+1..nc do
             β ← α(1– d^c_ij/min(x^c_i,x^c_j));
             for any k in 1..n–1 do
                  if Jk ⊂ H(I^c_i ∪ I^c_j) then αk ← max{αk,β};
                  endif;
             endfor;
        endfor;
   endfor;
   α ← max {αk}k∈1..n–1 ∩ [0,α);
   c ← c+1;

endwhile;
```

Fig. 12. Algorithm for the fuzzification of a longitudinal section I. The symbol $H(I^c_i \cup I^c_j)$ denotes the convex hull of $I^c_i \cup I^c_j$. The indexing is chosen such that the segments I^c_i and I^c_{i+1} are consecutive in I. The algorithm increases the membership degrees α_k associated with the open intervals $J_k = H(I^0_k \cup I^0_{k+1}) - I^0_k \cup I^0_{k+1}$ (initially, all α_k values are zero).

3.3 Coupling F-Histograms with Allen Relations

Consider an Allen relation r and the longitudinal sections $A_\theta(v)$ and $B_\theta(v)$ of some objects A and B. We are now able to define the value $F_r(\theta,A_\theta(v),B_\theta(v))$ (see the introductory paragraph of Section 3). If $A_\theta(v)=\varnothing$ or $B_\theta(v)=\varnothing$ then $F_r(\theta,A_\theta(v),B_\theta(v))$ is naturally set to 0. Assume $A_\theta(v)\neq\varnothing$ and $B_\theta(v)\neq\varnothing$. Assume $A_\theta(v)$, $B_\theta(v)$ and r have been fuzzified as described in Sections 3.1 and 3.2. There exists a tuple $(\alpha_0,\alpha_1,\ldots,\alpha_c)$ of real numbers such that $\alpha_0=0<\alpha_1<\alpha_2<\ldots<\alpha_c=1$ and $\{\alpha_k\}_{k\in 0..c} = \{\mu_{A\theta(v)}(M)\}_{M\in\Delta\theta(v)} \cup \{\mu_{B\theta(v)}(M)\}_{M\in\Delta\theta(v)}$ (the set of all membership values in the fuzzy sections $A_\theta(v)$ and $B_\theta(v)$). For any k in 1..c, there exists one set $\{I^k_i\}_{i\in 1..m_k}$ of mutually disjoint segments such that: $\alpha_k\, A_\theta(v) = \cup_{i\in 1..m_k} I^k_i$. Likewise, there exists one set $\{J^k_i\}_{i\in 1..n_k}$ of segments such that: $\alpha_k\, B_\theta(v) = \cup_{i\in 1..n_k} J^k_i$. For any i in $1..m_k$, the length of I^k_i is denoted by x^k_i. For any i in $1..n_k$, the length of J^k_i is denoted by z^k_i. The value $F_r(\theta,A_\theta(v),B_\theta(v))$ is defined as follows:

$$F_r(\theta,A_\theta(v),B_\theta(v)) = \Sigma_{k\in 1..c}\, \Sigma_{i\in 1..m_k}\, \Sigma_{j\in 1..n_k}\, [x^k_i\, z^k_j\, (\alpha_k-\alpha_{k-1})]\; r(I^k_i,J^k_j). \qquad (1)$$

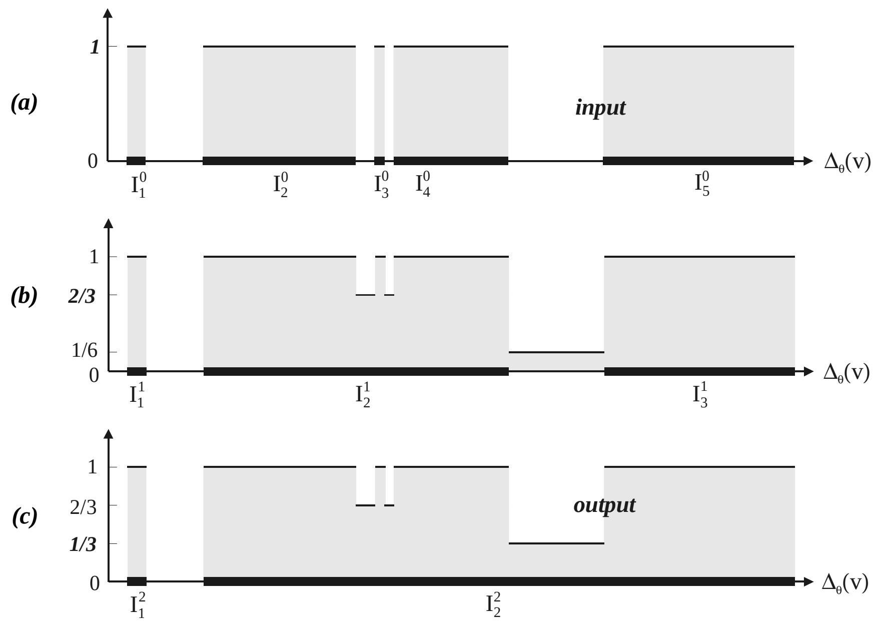


Fig. 13. Fuzzification of a longitudinal section I using the algorithm given in Section 3.2. Here, I is the union of five segments (n=5). Its membership function μ_I is plotted in *(a)*. We have: $x_1^0=1$ (length of I_1^0), $x_2^0=8$, $x_3^0=1/2$, $x_4^0=6$, $x_5^0=10$, $d_{12}^0=3$ (distance between I_1^0 and I_2^0), $d_{23}^0=1$, $d_{34}^0=1/2$, $d_{45}^0=5$. At the end of the first iteration of the `while` loop, μ_I is as shown in *(b)*. It has been modified because of two pairs of segments: (I_2^0, I_4^0) and (I_4^0, I_5^0). At the end of the second iteration, μ_I is as shown in *(c)*. It has been modified again, because of (I_2^1, I_3^1). The third and last iteration does not bring any changes. The fuzzified longitudinal section is therefore defined by the membership function plotted in *(c)*.

The issues raised by Figs. 8 to 10 are solved. For instance, since $r(I_i^k, J_j^k)$ is weighted by x_i^k and z_j^k, the emergence of a segment as in Fig. 10 has no significant impact on $F_r(\theta,A_\theta(v),B_\theta(v))$. Fig. 14 shows that the emergence of a hole in a segment has no real impact either. Small changes in the longitudinal sections do not affect $F_r(\theta,A_\theta(v),B_\theta(v))$ significantly. Continuity is satisfied and, hence, robustness is achieved. The F_r–histogram associated with (A,B) is as defined in Section 2.2:

$$F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta,A_\theta(v),B_\theta(v))\,dv. \qquad (2)$$

Remember that the issue raised by Fig. 9 has led us not to consider a longitudinal section a set of independent segments or points (Section 3.2). As a result, the F_r–histograms are neither f-histograms nor φ-histograms. Also note that the sum $\Sigma_{r\in\mathcal{A}} F_r(\theta,A_\theta(v),B_\theta(v)) = \Sigma_{k\in 1..c}\,\Sigma_{i\in 1..m_k}\,\Sigma_{j\in 1..n_k}\,[x_i^k\; z_j^k\,(\alpha_k-\alpha_{k-1})]$ does not depend on any Allen relation. Therefore:

$$\Sigma_{r\in\mathcal{A}} F_r^{AB}(\theta) = \Sigma_{r\in\mathcal{A}} \int_{-\infty}^{+\infty} F_r(\theta,A_\theta(v),B_\theta(v))\,dv = \int_{-\infty}^{+\infty} \Sigma_{r\in\mathcal{A}} F_r(\theta,A_\theta(v),B_\theta(v))\,dv$$

does not depend on any Allen relation either. Its value, however, is difficult to interpret. Let us redefine $F_r(\theta,A_\theta(v),B_\theta(v))$ this way [2] :

$$F_r(\theta,A_\theta(v),B_\theta(v)) = \frac{x+z}{w} \Sigma_{k\in 1..c} \Sigma_{i\in 1..m_k} \Sigma_{j\in 1..n_k} [x_i^k\, z_j^k\, (\alpha_k - \alpha_{k-1})]\; r(I_i^k, J_j^k)\,, \qquad (3)$$

where $x = \Sigma_{i\in 1..m_c} x_i^c$, $z = \Sigma_{j\in 1..n_c} z_j^c$, and $w = \Sigma_{k\in 1..c} \Sigma_{i\in 1..m_k} \Sigma_{j\in 1..n_k} [x_i^k\, z_j^k (\alpha_k - \alpha_{k-1})]$. We now have $\Sigma_{r\in\mathcal{A}} F_r(\theta,A_\theta(v),B_\theta(v)) = x+z$, and the value $\Sigma_{r\in\mathcal{A}} F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} \Sigma_{r\in\mathcal{A}} F_r(\theta,A_\theta(v),B_\theta(v))\,dv$ is the total area of the subregions of A and B that are “facing” each other in direction θ (Fig. 15). In other words, $\Sigma_{r\in\mathcal{A}} F_r^{AB}(\theta)$ tells us to what extent the objects are involved in some spatial relationships along direction θ. If this information is judged to be unimportant, the F_r–histograms, of course, can be normalized. Let us denote by $\lceil F_r^{AB} \rfloor$ the histogram F_r^{AB} after normalization. $\lceil F_r^{AB} \rfloor$ is defined by [2] :

$$\forall \theta \in \mathbb{R},\ \lceil F_r^{AB} \rfloor(\theta) = F_r^{AB}(\theta) \,/\, \Sigma_{\rho\in\mathcal{A}} F_\rho^{AB}(\theta). \qquad (4)$$

For a given oriented line $\Delta_\theta(v)$, the Allen relations define a crisp 13-partition of the set of pairs of segments on $\Delta_\theta(v)$. For a given direction θ, the normalized F_r–histograms define a fuzzy 13-partition of the set of all object pairs, and each class of the partition corresponds to an Allen relation.

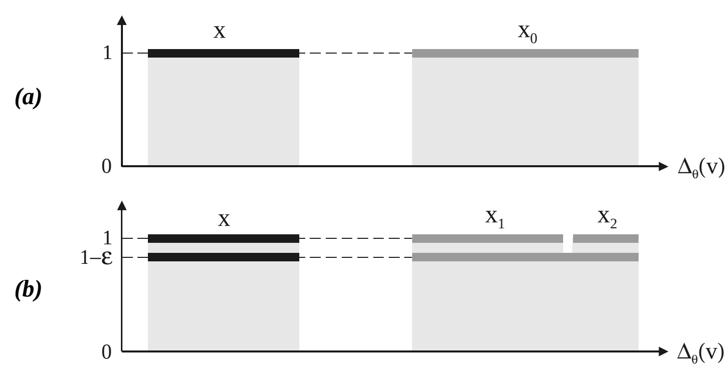


Fig. 14. In *(a)*, a missing pixel in the middle of one segment would not have much impact on $F_>(\theta,A_\theta(v),B_\theta(v))$. The way fuzzy relations are weighted (Eq. 1), combined with the way longitudinal sections are fuzzified, allow continuity to be satisfied. *(a)* $F_>(\theta,A_\theta(v),B_\theta(v)) = xx_0$. *(b)* $F_>(\theta,A_\theta(v),B_\theta(v)) = xx_1\varepsilon + xx_2\varepsilon + xx_0(1-\varepsilon) = x(x_1+x_2)\varepsilon + xx_0(1-\varepsilon) \approx xx_0\varepsilon + xx_0(1-\varepsilon) = xx_0$.

---

[2] In Eqs. 3 and 4 we agree that a fraction is 0 if its denominator is 0.

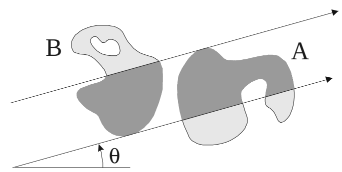


Fig. 15. The value $\Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\theta)$ is easy to interpret and gives useful information. In this example, $\Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\theta)$ is the total area of the two dark gray regions.

4 Experiments

In practice, of course, only a finite set of directions θ is considered. For the experiments described in this section, 360 directions were processed (i.e., the angle increment was 1 degree). All objects were stored in raster form. The computation of an F-histogram value, $F^{AB}(\theta)$, is achieved by partitioning the objects into longitudinal sections, i.e., into sets of adjacent pixels (Matsakis 1998; Matsakis and Wendling 1999). The generation of these sections is based on the rasterization of a pencil of parallel lines (Fig. 2) by means of Bresenham's algorithm in integer arithmetic, which is commonly circuit-coded in visualization systems. The handling of a pair of objects then comes down to the handling of pairs of longitudinal sections, as described by Eq. 3. Note that, in a given image, all pairs of objects can be processed simultaneously. Moreover, F-histogram computation is highly parallelizable.

A grayscale value is associated with each Allen relation (Fig. 16(a)). The referent, B, is always shown in dark gray and the argument, A, in light gray (Fig. 16(b)). The thirteen F_r–histograms that represent the extracted directional and topological relationship information are plotted in the same diagram (Fig. 16(c)). The topological relationships along direction θ (on the X-axis) are described by the vector composed of the thirteen $F_r^{AB}(\theta)$ values (on the Y-axis). Usually, most of these values are zero. The histograms are arranged in "layers." For a given θ, the total height of the layers (i.e., $\Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\theta)$) represents an area, as described in Section 3.3 and Fig. 15. It tells us to what extent the objects are involved in some spatial relationships along θ. The thirteen normalized F_r–histograms can be plotted in the same way (Fig. 16(d)). Figs. 16 and 17 show two object pairs and the corresponding diagrams. Fig. 16(d) and Fig. 17(c) illustrate well the symmetric nature of the histograms. For any θ, we have:

$$F_{>}^{AB}(\theta) = F_{<}^{AB}(\theta+\pi) \;\text{ and }\; F_{mi}^{AB}(\theta) = F_{m}^{AB}(\theta+\pi)$$
$$\text{and }\; F_{oi}^{AB}(\theta) = F_{o}^{AB}(\theta+\pi) \;\text{ and }\; F_{si}^{AB}(\theta) = F_{fi}^{AB}(\theta+\pi) \;\text{ and }\; F_{f}^{AB}(\theta) = F_{s}^{AB}(\theta+\pi).$$

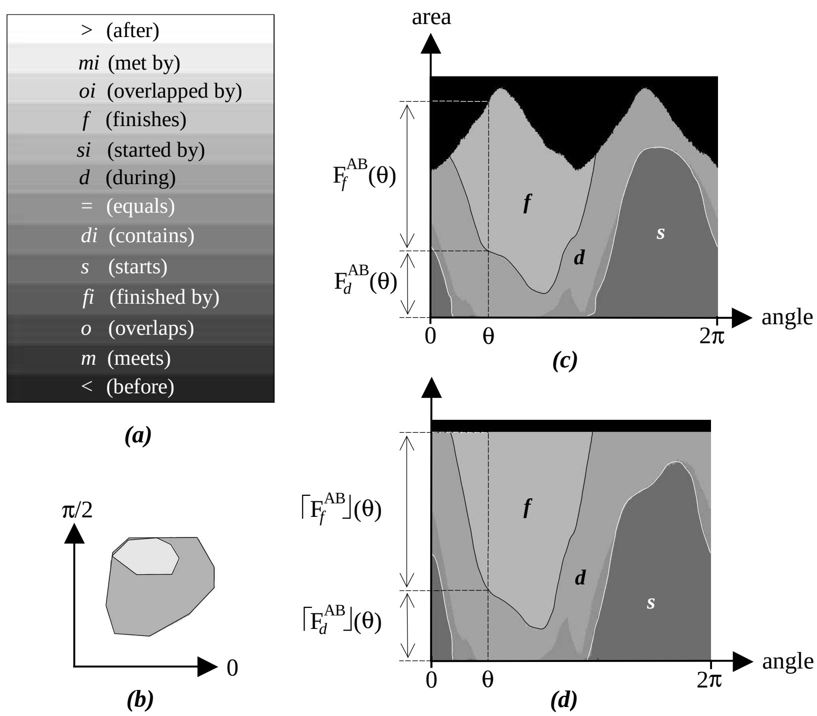


Fig. 16. *(a)* Allen relations and attached grayscale values. *(b)* A pair of objects. *(c)* Corresponding F_r–histograms. *(d)* Normalized F_r–histograms.

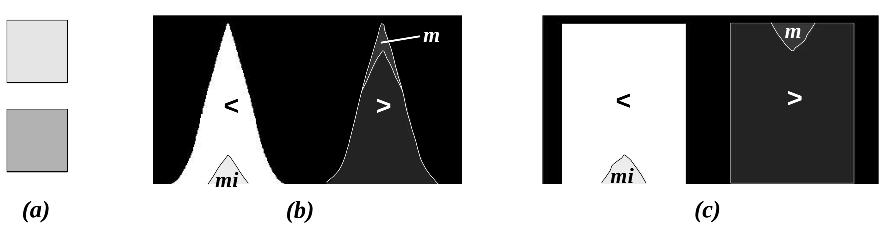


Fig. 17. *(a)* A pair of objects. *(b)* Corresponding F_r–histograms. *(c)* Normalized F_r–histograms.

The first series of experiments illustrates how the fuzzy relations defined in Section 3.1 are interconnected. It also demonstrates how the prominence of different relations waxes and wanes with the change of distance between the objects. In Fig. 18(a), the objects are quite far apart, and only the relations *before* and *after* are present. As the distance shortens, Fig. 18(b) and Fig. 18(c), *meets* and *met by* appear and become more and more prominent, while *before* and *after* decrease in their importance. Finally, when the objects touch, Fig. 18(d), *meets* and *met by* perfectly describe the scene.

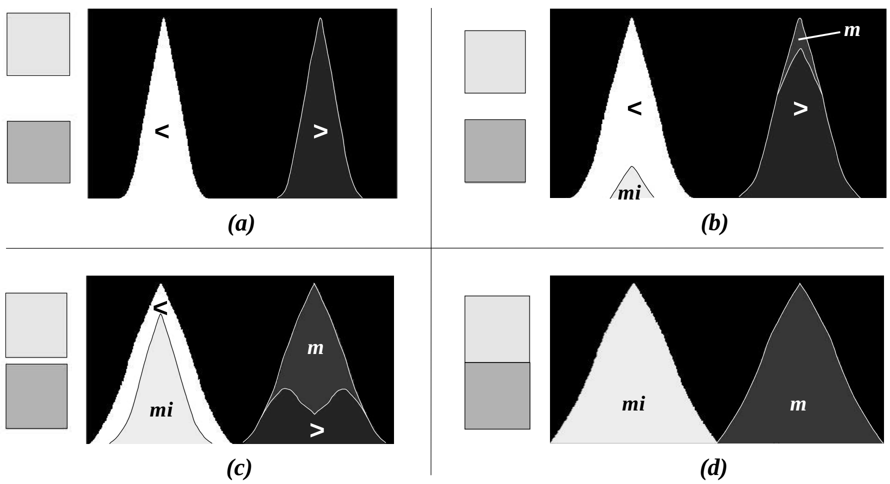


Fig. 18. First series of experiments. Interconnection of fuzzy relations and sensitivity to distance. *(a)* Objects far apart. Relations *before* and *after*. *(b)* Objects closer together. Relations *before*, *after*, *meets* and *met by*. *(c)* Objects very close together. *before* and *after* are less prominent, *meets* and *met by* are more. *(d)* Objects touching. Relations *before* and *after* disappear.

In the second series of experiments (Fig. 19), we examine the relations between two convex objects A and B as A moves towards B, intersects it and, finally, goes through it. In Fig. 19(a) and Fig. 19(b), the only relations between A and B are *before* and *after*. As A moves towards B, the support of the two relations becomes wider. Once A intersects B, the relations become more complex and are mainly represented by the symmetric pairs *before – after, meets – met by* and *overlaps – overlapped by* (Fig. 19(c)). In directions close to horizontal, there are also small contributions from *equals* and its conceptual neighbors *contains* and *during*. Once A is completely in B (Fig. 19(d)), but still very close to its top edge, the relations *finishes* (when viewed from the bottom) and *starts* (when viewed from the top) become prominent and the relation *during* is consistently present. Once the object A moves further towards the center of B, *during* becomes by far the most important relation (Fig. 19(e)); *finishes* and *starts* occur only briefly, in the directions where the distances between the edges of A and B are the smallest.

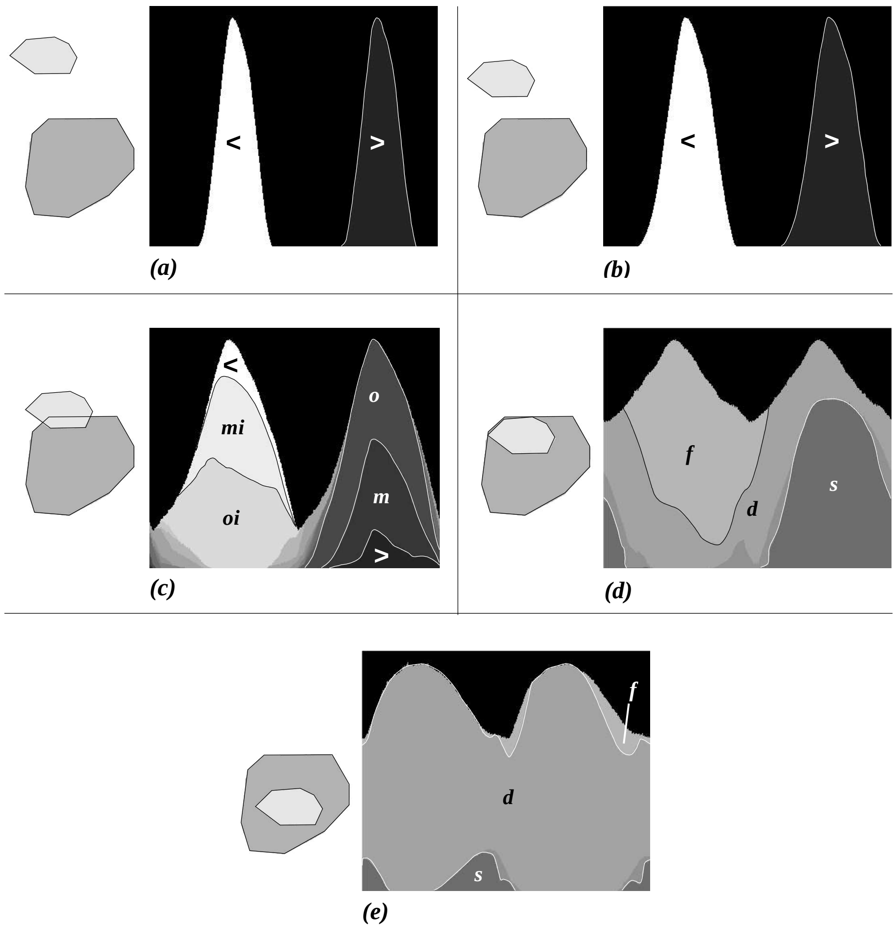


Fig. 19. Second series of experiments. Complex relation changes in a dynamic scene between convex objects. The same scene is considered in (Malki et al. 2002).

The final set of experiments involves concave objects. It illustrates how the F_r–histograms allow more complex topological relationships to be described and differentiated. In Fig. 20(a), a convex object, B, is partially surrounded by a concave object, A. The diagram shows that the relations *before* and *after* coexist equally in the horizontal directions (B is between equidistant, equally thick "arms"). As the direction θ changes, *before* increases and then decreases in prominence, followed in its behavior by *after*. The twin "peaks" in the diagram (for both *before* and *after*) occur when θ passes through B and the arms of A (diagonal directions), whereas the "valleys" occur when θ passes through B and the "body" of A (vertical directions). In Fig. 20(b), the concave object is asymmetrical, and the histograms are less regular. In the horizontal directions, *before* and *after* do not coexist any longer. Note that small contributions from *meets* and *met by* appear in both experiments due to the proximity of the objects. Also note the complete absence of the pair *overlaps – overlapped by*.

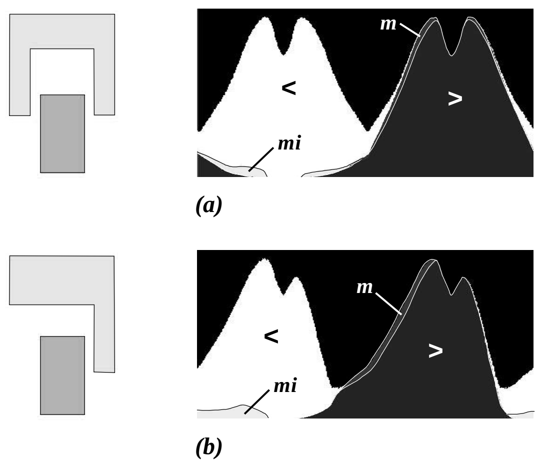


Fig. 20. Third series of experiments. Handling of concave objects. The same pairs are considered in (Petry et al. 2002). *(a)* The referent is partially surrounded by the argument. *(b)* The referent is surrounded to a smaller degree.

5 Conclusion

The F-histogram is a powerful generic quantitative representation of the relative position between two 2D objects. In this chapter, we have designed a set of thirteen histograms that constitutes a valuable tool for extracting directional and topological relationship information. Imprecision is handled and robustness achieved through fuzzy set theoretic approaches. For any direction in the plane, the F-histograms introduced here define a fuzzy 13-partition of the set of all pairs of objects, where each class of the partition corresponds to an Allen relation. The considered objects are not necessarily convex, nor connected, and they may have holes in them. We have shown that the F-histograms associated with a given pair of objects carry lots of relationship information. For instance, an ambiguity index can be calculated to assess the complexity of the topological relationships along any direction. If so desired, only the Allen relation that represents these relationships the best can be kept (defuzzification). Alternatively, two Allen relations can be kept—the most prominent—and weighed by their corresponding membership degrees. The number of directions to be processed can be chosen according to needs, interests and constraints (e.g., accuracy, computational efficiency). It can be as low as two (horizontal and vertical directions, like for MBRs) and as large as a few hundred (e.g., the increment step of 1° chosen for our experiments). Since directions are handled independently from each other, additional ones can be considered in a second stage, depending on the case in hand (dynamic refinement). The direction for which the ambiguity index is minimum can be searched for. Spatial relationships can be compared from one pair of objects to another, using similarity or distance measures between the vectors of membership degrees in all considered directions. These are avenues that we intend to explore in future work.

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