Fuzzy Object Localization Based on Directional (and Distance) Information

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***Abstract*—A directional spatial relationship to a reference object (e.g., "east of the post office") can be represented by a spatial template, i.e., a fuzzy subset of the Euclidean space. For each point of the space, the template indicates to what extent the relationship holds. The objects for which the relationship holds best can then be located. In previous work, we discussed the case of crisp 2D objects in raster form. We introduced a new algorithm for directional spatial template computation, which is faster, gives better results and is more flexible than its competitors. The present paper continues this line of research. The algorithm is extended to handle fuzzy objects and embed distance information. In existing models, only angular deviation is taken into account. Spatial distance, however, also contributes in shaping directional templates.**

I. Introduction

SPACE plays a fundamental role in human cognition. In everyday situations, it is often viewed as a construct induced by spatial relationships, rather than as a container that exists independently of the objects located in it. Spatial relationships, therefore, have been thoroughly investigated in many disciplines, including cognitive science, psychology, linguistics, geography and artificial intelligence. They act as a connecting link between visually perceived data and natural language, and an important part of research naturally deals with two types of tasks: those related to the translation of visual information into linguistic expressions (e.g., automatic digital image analysis and description), and those related to the translation of linguistic expressions into visual information (e.g., query processing in spatial database systems). In this paper, we focus on directional (also called projective) relationships (e.g., front, south, above). The past ten to fifteen years have seen significant advancements in the development of mathematical and computational models of these relationships [1] [2] [3] [4] [5]. Tasks of the first type require from such models the capability to identify which relationships hold best between any two objects. Tasks of the second type require different capabilities. Given a directional relationship to a reference object (e.g., "east of the post office"), the models should be able to identify the objects for which the relationship holds best, and also to distinguish regions where it holds from regions where it does not hold. These regions, of course, blend into one another. Whether implicitly or explicitly, they are usually represented by a fuzzy subset of the Euclidean space. Different names can be found in the literature (e.g., "fuzzy landscape" [6], "spatial template" [7], "applicability structure" [8], "potential field" [9]). Here, we will use the term "directional spatial template" (or "template", for short). For each point of the space, the template indicates to what extent the relationship holds. Directional relationships defy precise definitions. The idea that fuzzy set theory should be applied was suggested more than thirty years ago [10] and has since been widely accepted.

There exists a very simple and yet cognitively plausible way to model a template without sacrificing the geometry of the reference object (i.e., the object is not approximated through its centroid or minimum bounding rectangle). Computationally, however, exact calculation of the model in case of 2D raster data is prohibitively expensive, and a tractable approximation algorithm was proposed in [6]. We recently presented another approximation algorithm, which is faster, gives better results and is more flexible [11]. On the downside, only crisp objects were considered. This is a limitation, since image regions are often represented as spatial fuzzy sets, with the purpose of taking into account different types of imprecision. Possible sources of imprecision include our ignorance (e.g., extent of a mineral deposit), intrinsic vagueness (e.g., marshlands), image acquisition (e.g., spatial resolution) and processing (e.g., filtering procedures). In Section III, we show that our algorithm can easily be extended to handle fuzzy objects, while keeping its advantages. Although a directional template depends mainly on angular deviation, spatial distance to the reference object also contributes in its shaping [7] [12] [13]. The only models that do not ignore distance information, however, approximate the object through its centroid or minimum bounding rectangle (see, e.g., [8]). In Section IV, the issue is examined, and distance information is embedded into our model. Conclusions and directions of future work are given in Section V. First, in Section II, we introduce some notations and briefly review two important concepts.

II. Basic Templates and F-Templates

A. Notations

In this paper, $\Re$ denotes the set of real numbers, $\Re_+$ the set of non-negative real numbers (≥0) and $\Re_+^*$ the set of positive real numbers (>0). The symbol μ denotes a mapping from $\Re$ into [0,1], periodic with period 2π, even, decreasing on $[0,\pi]$, and such that $\mu(0)=1$ and $\mu(\pi/2)=0$. See Fig. 1. The

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symbol $\mathcal{P}$ denotes the Cartesian plane. For any two points q and p in $\mathcal{P}$, with $q \neq p$, the expression $\angle(q,p)$ represents the direction of the vector $\overrightarrow{qp}$. It is a value that belongs to the interval $[-\pi,\pi)$. An *object* is a non-empty subset of $\mathcal{P}$. For any object B, any θ in $[0,\pi)$ and any p in $\mathcal{P}$, the symbol $B_p(\theta)$ denotes the intersection of B with the line that runs in direction θ and passes through p. This intersection is a *longitudinal section* of B (or *section*, for short). See Fig. 2. The set of all possible longitudinal sections is denoted by $\mathcal{L}$. Consider some section J, i.e., some element J of $\mathcal{L}$. There exists a line Δ_J that includes J. This line runs in some direction $\theta_J \in [0,\pi)$. Assume $p \in \Delta_J$. The symbols $\Delta^-_{J,p}$ and $\Delta^+_{J,p}$ denote the two half-lines such that

$$\Delta^-_{J,p} \cup \Delta^+_{J,p} = \Delta_J \ \text{ and } \ \Delta^-_{J,p} \cap \Delta^+_{J,p} = \{p\} \qquad (1)$$

$$\forall q \in \Delta^-_{J,p} - \{p\}, \ \angle(q,p) = \theta_J - \pi \qquad (2)$$

$$\forall q \in \Delta^+_{J,p} - \{p\}, \ \angle(q,p) = \theta_J \qquad (3)$$

See Fig. 3. The notations above hold whether B is crisp or fuzzy. Note that a section J of a fuzzy object is a fuzzy subset of the crisp line Δ_J.

B. Basic Templates

In this section, A and B denote crisp objects. If you were told that A was perfectly (or somewhat, or not at all) in direction δ (e.g., west, above-right) of B, where in space would you look for A? Cognitive experiments suggest that you would mentally build a spatial template [7] [12] [13]. Using essentially angular deviation, you would partition the

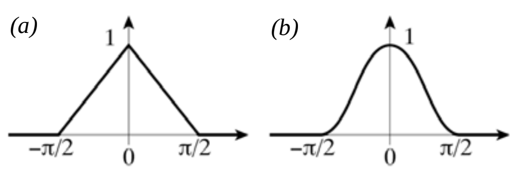

Fig. 1. Two possible functions μ.

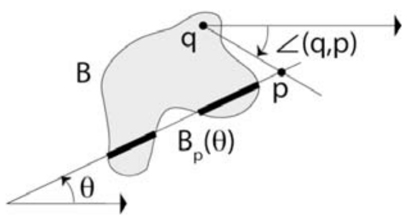

Fig. 2. Points, angles, objects, and longitudinal sections.

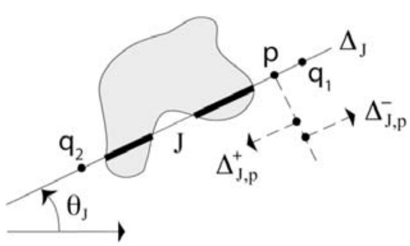

Fig. 3. The line Δ_J runs in direction θ_J and includes the section J. The point p splits Δ_J into two half-lines, $\Delta^-_{J,p}$ and $\Delta^+_{J,p}$. The point q_1 in $\Delta^-_{J,p}$ is such that $\angle(q_1,p) = \theta_J - \pi$. The point q_2 in $\Delta^+_{J,p}$ is such that $\angle(q_2,p) = \theta_J$.

space into regions where "in direction δ of B" holds to various degrees. It therefore makes sense to represent this template by a mapping from $\mathcal{P}$ into $\Re$, such as $S^{\delta B}$.

$$\forall\, p \in \mathcal{P}, \ S^{\delta B}(p) = \sup_{q \in B - \{p\}} \mu(\angle(q,p) - \delta) \qquad (4)$$

$S^{\delta B}$ defines a fuzzy subset of the Cartesian plane, called the *basic directional spatial template* induced by B in direction δ. As an example, Fig. 4b shows the basic template induced by some reference building in direction north. The brighter the area, the higher the membership value $S^{\delta B}(p)$, i.e., the more it is considered that the area is north of the reference building. In case of raster data, the algorithm that corresponds to Eq. 4 is straightforward. Computationally, however, it is prohibitively expensive, and a tractable approximation algorithm was proposed in [6]. Note that $S^{\delta B}$ allows the proposition "A is in direction δ of B" to be readily assessed for any object A. The degree of truth of this proposition can be set to, e.g., $\sup_{p \in A} S^{\delta B}(p)$, or $\inf_{p \in A} S^{\delta B}(p)$. These two values correspond to the most optimistic and most pessimistic points of view (Fig. 4c). They can also be interpreted as a possibility degree and a necessity degree [6].

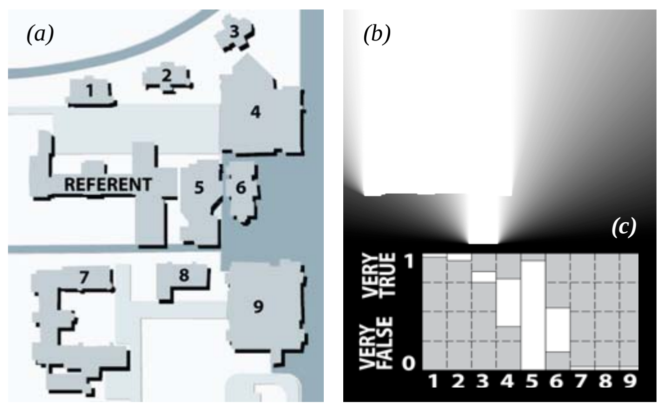

Fig. 4. *(a)* Campus map. *(b)* Basic spatial template: show me where the north is (relative to the reference building). *(c)* Are the buildings 1 to 9 north of the reference one? The white bars represent all possible points of view.

C. F-Templates

F-templates were introduced in [11] as a concept dual to that of the F-histogram (described much earlier in [14] [4]). Let δ be a direction and B a crisp object. An *F-template* induced by B in direction δ is a mapping $F^{\delta B}$ from $\mathcal{P}$ into $\Re$. In "$F^{\delta B}$", the letter "F" denotes a function from $\mathcal{P} \times \Re \times \mathcal{L}$ into $\Re$. (The letter "F" in the expression "F-template", however, is not dissociable from the word "template", and does not refer to any specific function.) The value $F^{\delta B}(p)$ is a combination of the $F(p,\delta,B_p(\theta))$ values, for all θ (Fig. 5). In the rest of this paper

$$\forall\, p \in \mathcal{P}, \ \ F^{\delta B}(p) = \sup_{\theta \in [0,\pi)} F(p,\delta,B_p(\theta)) \qquad (5)$$

Now, consider the function F defined as follows, for any crisp section J, direction δ, and point p aligned with J.

$$\begin{array}{ll}\text{If } J=\varnothing \text{ or } J=\{p\} \text{ then } F(p,\delta,J)=0, & (6)\\ \text{else if } J\subset\Delta^{-}_{J,p} \text{ then } F(p,\delta,J)=\mu((\theta_J-\pi)-\delta), & (7)\\ \text{else if } J\subset\Delta^{+}_{J,p} \text{ then } F(p,\delta,J)=\mu(\theta_J-\delta), & (8)\\ \text{else } F(p,\delta,J)=\max\{\,\mu((\theta_J-\pi)-\delta)\,,\mu(\theta_J-\delta)\,\}. & (9)\end{array}$$

The corresponding F-template $F^{\delta B}$ is equal to the basic directional spatial template $S^{\delta B}$. In case of raster data, Eqs. 5 to 9 lead to a new approximation algorithm, which is faster and gives better results than the algorithm presented in [6]. Details can be found in [11].

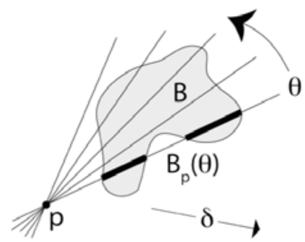

Fig. 5. F-templates. Each line gives an $F(p,\delta,B_p(\theta))$ value. $F^{\delta B}(p)$ is a combination of the $F(p,\delta,B_p(\theta))$ values, for all θ.

III. Case of Fuzzy Objects

A. Equations

Let δ be a direction and B a fuzzy object. The basic directional spatial template $S^{\delta B}$ is naturally redefined as follows, where t denotes a fuzzy conjunction (i.e., a t-norm) and B(q) the membership degree of q in B.

$$\forall\, p \in \mathcal{P},\ \ S^{\delta B}(p) = \sup_{q\in\mathcal{P}-\{p\}} t\,(\,B(q)\,,\,\mu(\angle(q,p)-\delta)\,) \qquad (10)$$

This is nothing new (see, e.g., [6]). Note that Eqs. 4 and 10 are consistent, i.e., if the object B is crisp, then Eq. 10 comes down to Eq. 4. Now, consider the function F defined as follows, for any point p, direction δ, and fuzzy longitudinal section J. Again, t denotes a fuzzy conjunction and J(q) the membership degree of q in J.

$$\begin{aligned}F(p,\delta,J) = \max\{ &\sup_{q\in\Delta^{-}_{J,p}-\{p\}} t\,(\,J(q)\,,\,\mu((\theta_J-\pi)-\delta)\,)\,,\\ &\sup_{q\in\Delta^{+}_{J,p}-\{p\}} t\,(\,J(q)\,,\,\mu(\theta_J-\delta)\,)\,\}\end{aligned} \qquad (11)$$

Equation 11 and Eqs. 6 to 9 are consistent. They give the same value if J is crisp. Moreover, it is easy to show that the directional F-template $F^{\delta B}$ defined by Eqs. 5 and 11 is equal to the basic directional template $S^{\delta B}$ as in Eq. 10.

B. Implementation

In case of 2D raster data, the algorithm that corresponds to Eq. 10 is straightforward but computationally expensive, since its complexity is quadratic in the number of pixels in the image. The directional F-template $F^{\delta B}$ defined as in Eqs. 5 and 11 is computed very much like F-histograms, using the duality between the two concepts. Let us describe the principle of the algorithm. First, $F^{\delta B}$ is initialized to 0, and a finite set of reference directions evenly distributed in the Cartesian plane is selected (e.g., {0°,1°, … ,359°}). Then, for each θ of that set, the image is partitioned into rasterized lines that run in direction θ. When growing such a line, the pixels q_1, q_2, …, q_k are successively encountered. The values $F^{\delta B}(q_i)$ are updated as follows:

```
maxMembership ← 0
FOR all i in 1..k DO
| IF B(qi)>maxMembership THEN maxMembership ← B(qi)
| FδB(qi) ← max { FδB(qi) , t(μ(θ−δ), maxMembership) }
```

Besides this update procedure, the algorithm is the same as the algorithm presented in [11] (case of crisp objects). The reader can therefore refer to [11] for details on, e.g., optimization and template initialization (which is actually not $F^{\delta B} \leftarrow 0$). The complexity of the algorithm is linear in the number of pixels in the image and in the number of reference directions.

As an example, Fig. 6 shows a fuzzy object and the F-template it induces in direction 180° (left). As stressed in [6]—where Fig. 6a is taken from—directional spatial templates induced by fuzzy structures can be of use in model-based pattern recognition. The object in Fig. 6a represents the left ventricle of a human brain, and was obtained through fuzzy segmentation of a magnetic resonance image.

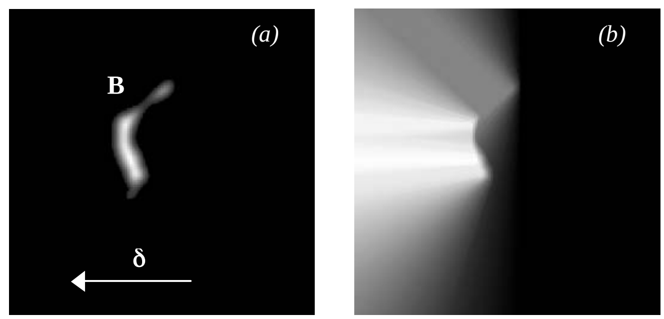

Fig. 6. *(a)* A fuzzy reference object. *(b)* The F-template it induces in direction 180° (left).

C. Comparative experimental study

In this section, the basic directional spatial template $S^{\delta B}$ (see Eq. 10) is compared with its approximations $F^{\delta B}$ (the F-template computed as in Eqs. 5 and 11) and $M^{\delta B}$ (the fuzzy landscape computed as in [6], using a *m*orphological approach). Figures 7 and 8 illustrate well our findings. Basic directional spatial templates can be approximated faster and better by F-templates. By simply adjusting the number of reference directions, users can finely control the balance between quality and processing time (Fig. 7). Also note that F-templates induced by the same reference object B in different directions δ_1, δ_2, δ_3, … can be batch-processed [11]. Experiments were conducted on a 1.8GHz P4 with 1024MB memory, running *Windows XP*. The implementation language was *C++*. In Eqs. 10 and 11, the function μ was as in Fig. 1a, and the fuzzy conjunction t was the algebraic product (i.e., as in [6] for the computation of $M^{\delta B}$). The images were 8-bit images.

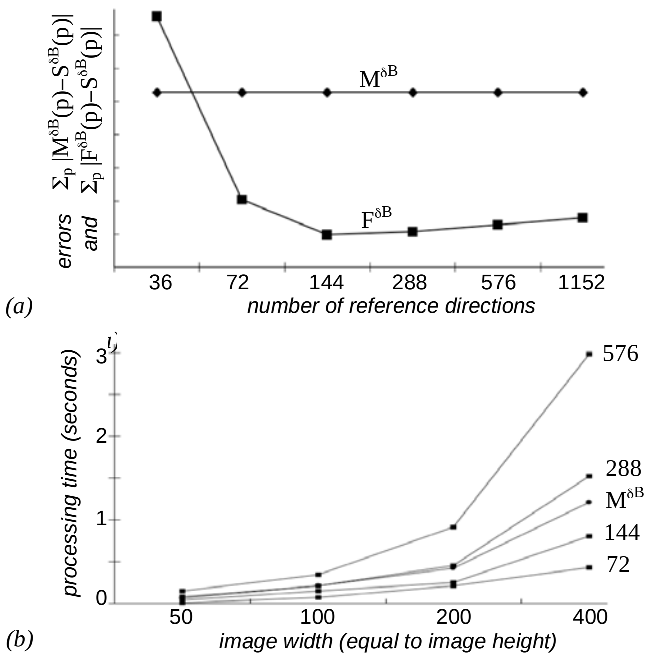

Fig. 7. ***(a)*** **Quality analysis.** ***(b)*** **Efficiency analysis. In** ***(b)*****, processing time for $F^{\delta B}$ depends on the number of reference directions (72, 144, 288 or 576). Processing time for $S^{\delta B}$ is about 10 seconds when the image width is 200, and about 150 seconds when the width is 400.**

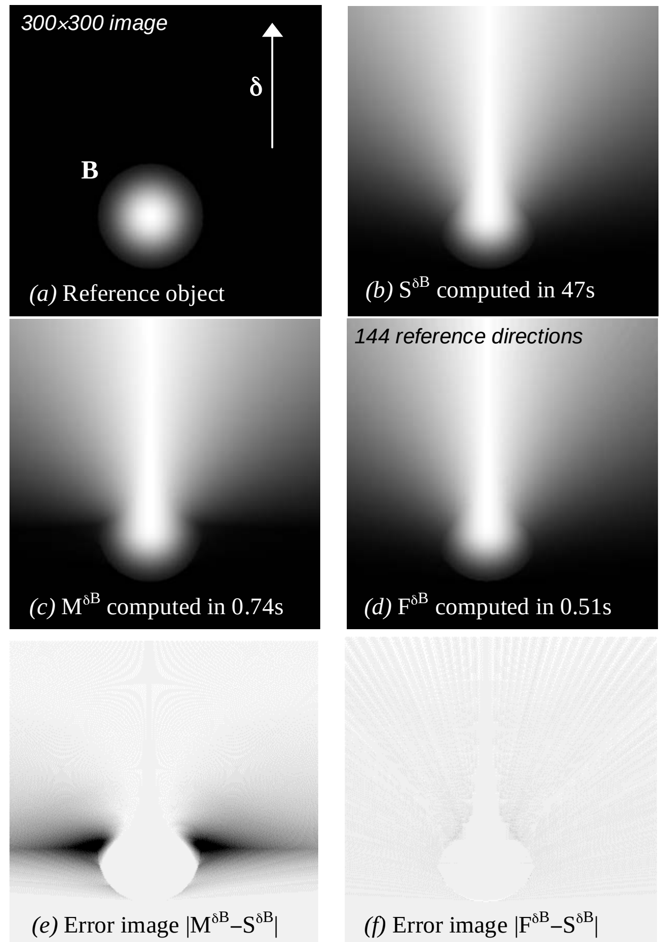

Fig. 8. Comparative example. The basic template $S^{\delta B}$ is approximated by the fuzzy landscape $M^{\delta B}$ (see [6]) and the F-template $F^{\delta B}$. The error images are contrast-enhanced. The darker, the higher the error. The average error of $M^{\delta B}$ is about five times the average error of $F^{\delta B}$.

IV. Embedding Distance Information

A. Reason and principle

Cognitive experiments show that a directional spatial template depends mainly on angular deviation [7] [12] [13]. They also show, however, that distance contributes in shaping the template. For a given angular deviation, the membership degrees are not constant. They fluctuate slightly, depending on the distance to the reference object. Moreover, the fluctuation varies from one angular deviation to another. Finally, when sufficiently far from the object, all the membership degrees usually drop. For example, if you were told that the soccer ball was to the right of the bench, you would not look for it hundreds of feet from the bench. Distance information, therefore, can help improve the modeling of directional spatial templates.

Let $F^{\delta B}_{\mathrm{ANG}}$ be an F-template that models the relationship "in direction δ of B" based solely on angular deviation.

$$\forall p \in \mathcal{P},\ \ F^{\delta B}_{\mathrm{ANG}}(p) = \sup_{\theta \in [0,\pi)} F_{\mathrm{ANG}}(p,\delta,B_p(\theta)) \tag{12}$$

The function F_{ANG} can be defined as in Section III.A (Eq. 11). Now, assume we are able to model the relationship "close to B" by an F-template $F^{\delta B}_{\mathrm{DIST}}$.

$$\forall p \in \mathcal{P},\ \ F^{\delta B}_{\mathrm{DIST}}(p) = \sup_{\theta \in [0,\pi)} F_{\mathrm{DIST}}(p,\delta,B_p(\theta)) \tag{13}$$

$F^{\delta B}_{\mathrm{DIST}}(p)$ can be seen as the degree of truth of the proposition "p is close to B", and $F_{\mathrm{DIST}}(p,\delta,B_p(\theta))$ as the degree of truth of "p is close to $B_p(\theta)$". The two functions F_{ANG} and F_{DIST} allow us to define a third F-template $F^{\delta B}$, where $F = t(F_{\mathrm{ANG}}, F_{\mathrm{DIST}})$ and t denotes a fuzzy conjunction. For any p,

$$F^{\delta B}(p) = \sup_{\theta \in [0,\pi)} t(F_{\mathrm{ANG}}(p,\delta,B_p(\theta)), F_{\mathrm{DIST}}(p,\delta,B_p(\theta))) \tag{14}$$

$F^{\delta B}(p)$ represents a logical assessment of the proposition "p is in direction δ of B" based on both directional and distance information. Note that $t(F^{\delta B}_{\mathrm{ANG}}(p), F^{\delta B}_{\mathrm{DIST}}(p))$ is another possible assessment of the proposition. $t(F^{\delta B}_{\mathrm{ANG}}, F^{\delta B}_{\mathrm{DIST}})$, however, is not an F-template, and distance information is not really *embedded* into the model. The fusion of information from independent sources is a different problem, which will not be considered here. $F^{\delta B}_{\mathrm{ANG}}$ and $F^{\delta B}_{\mathrm{DIST}}$ have been introduced for the only purpose of facilitating the reading.

In Section IV.B, we assume the object B is crisp and we examine what is probably the simplest and most natural way to model the relationship "close to B". We show that this model can be seen as an F-template $F^{\delta B}_{\mathrm{DIST}}$, which gives us a first candidate for the function F_{DIST} in Eq. 14. We argue, however, that some properties of this function might not be desirable. Our analysis leads us to introduce, in Section IV.C, another candidate for F_{DIST}. The case of fuzzy objects is considered in Section IV.D and experimental results are presented in Section IV.E.

B. First proposition

According to Eq. 14, distance information can be embedded into our model—described by Eqs. 5 and 11—through a function F_{DIST}. In this section, a first candidate for F_{DIST} is presented. The distance between a point p and a crisp object B is usually defined as $\inf_{q\in B} pq$, where pq denotes the Euclidean distance between p and q. The degree of truth of the statement "p is close to B" could therefore be set to $h(\inf_{q\in B} pq)$, where h is a continuous, non-increasing mapping from $\Re_+$ onto [0,1]. The fuzzy subset of the Cartesian plane so defined can actually be seen as an F-template. In other words, it is possible to find a direction δ and a function F from $\mathcal{P}\times\Re\times\mathcal{L}$ into [0,1] such that:

$$\forall p\in\mathcal{P},\quad F^{\delta B}(p) = \sup_{\theta\in[0,\pi)} F(p,\delta,B_p(\theta)) = h(\inf_{q\in B} pq) \qquad (15)$$

We assume here that any (crisp) longitudinal section J is the union $\cup_{i\in 1\ldots n} J_i$ of a finite number of pairwise disjoint segments. If J is empty, n is 0. Consider a point p aligned with J. Let z_i be the length of the segment J_i and let y_i be the distance between p and J_i. Equation 15 holds (for any δ) when F is chosen such that

$$F(p,\delta,J) = h(\min_{i\in 1..n} g(z_i,y_i)) \qquad (16)$$

with

$$\forall (z,y)\in\Re_+^{\,2},\quad g(z,y) = y \qquad (17)$$

By convention, $\min_{i\in 1..0} g(z_i,y_i) = +\infty$ and $h(+\infty) = 0$. Note that Eq. 16 can be replaced with Eqs. 18 and 19 below. This is illustrated by Fig. 9, and will be useful in the next sections.

$$\begin{aligned}&(J\subset\Delta^-_{J,p} \text{ or } J\subset\Delta^+_{J,p})\\ \Rightarrow\ & F(p,\delta,J) = h(\min_{i\in 1..n} g(z_i,y_i))\end{aligned} \qquad (18)$$

$$\begin{aligned}&(J\not\subset\Delta^-_{J,p} \text{ and } J\not\subset\Delta^+_{J,p})\\ \Rightarrow\ & F(p,\delta,J) = \max\{\, F(p,\delta,J\cap\Delta^-_{J,p}),\ F(p,\delta,J\cap\Delta^+_{J,p}) \,\}\end{aligned} \qquad (19)$$

The function F above is a candidate for F_{DIST} in Eq. 14. There are two reasons, however, why one might not be happy with it. Consider $F^{\delta B}$ (as in Eq. 15). This spatial fuzzy set is not defined relative to the size of the object B. According to it, if John is close to the Olympic stadium when 100 feet from it, then Mary is close to her glasses when 100 feet from them. Moreover, a region of negligible size (e.g., a single pixel) can change $F^{\delta B}$ drastically (Fig. 10). Note that the same comment applies to all existing models of directional spatial templates (including our F-template model described by Eqs. 5 and 11). The first issue could be addressed by setting the distance unit to the radius of the disk whose area is the area of B. One might try to resolve the second issue by opening B beforehand—where "opening" should be understood in the context of mathematical morphology. In the next section, we design a function F that naturally answers both issues.

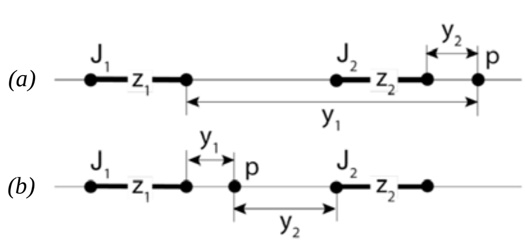

Fig. 9. ***(a)*** **$J=J_1\cup J_2$ is included in $\Delta^-_{J,p}$ or in $\Delta^+_{J,p}$. The degree of truth of "p is close to J" is $h(y_2)$, i.e., $h(\min\{y_1,y_2\})$ (Eq. 18).** ***(b)*** **$J=J_1\cup J_2$ is not included in $\Delta^-_{J,p}$ and is not included in $\Delta^+_{J,p}$. The degree of truth of "p is close to J" is $h(y_1)$, i.e., $\max\{h(y_1),h(y_2)\}$ (Eq. 19).**

C. Second proposition

Here, another candidate for the function F_{DIST} in Eq. 14 is presented. As in Section IV.B, the Greek letter δ denotes a direction, $J = \cup_{i\in 1\ldots n} J_i$ a crisp longitudinal section (empty if n is 0) and p a point aligned with J. The symbol z_i denotes the length of the segment J_i and the symbol y_i the distance between p and J_i. Our candidate, F, is defined by Eqs. 20 and 21 below. The value F(p,δ,J) can be seen as the degree of truth of the statement "p is close to J". In this paper, we assume it does not depend on δ, nor on θ_J.

$$\begin{aligned}&(J\subset\Delta^-_{J,p} \text{ or } J\subset\Delta^+_{J,p})\\ \Rightarrow\ & F(p,\delta,J) = h(\Sigma_{i\in 1..n}\, g(z_i,y_i))\end{aligned} \qquad (20)$$

$$\begin{aligned}&(J\not\subset\Delta^-_{J,p} \text{ and } J\not\subset\Delta^+_{J,p})\\ \Rightarrow\ & F(p,\delta,J) = \max\{\, F(p,\delta,J\cap\Delta^-_{J,p}),\ F(p,\delta,J\cap\Delta^+_{J,p}) \,\}\end{aligned} \qquad (21)$$

As expressed by Eq. 21, the second case comes down to the first one. In Eq. 20 (which should be compared with Eq. 18),

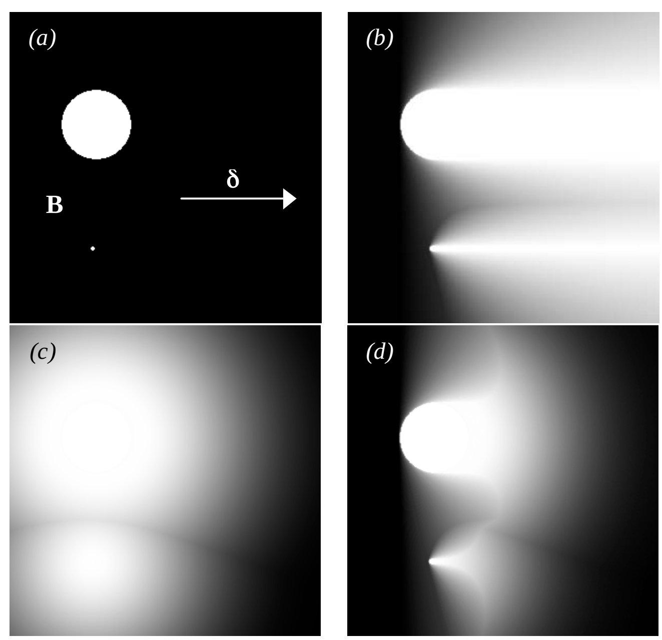

Fig. 10. ***(a)*** **B and δ.** ***(b)*** **$F^{\delta B}_{ANG}$, where F_{ANG} is defined as in Eqs. 6 to 9 (with t = min and μ as in Fig. 1a).** ***(c)*** **$F^{\delta B}_{DIST}$, where F_{DIST} is as in Eqs. 16 and 17 (with $h(x) = \exp[-(x/100)^2]$, for any $x\in\Re_+$).** ***(d)*** **$\min\{F^{\delta B}_{ANG}, F^{\delta B}_{DIST}\}$. The region of negligible size in the bottom-left part of *(a)* has a significant impact on *(b)*, *(c)* and *(d)*.**

[G1] g is a continuous mapping from $\Re_+ \times \Re_+^*$ into $\Re$.
[G2] $\forall k \in \Re_+^*, \forall z \in \Re_+, \forall y \in \Re_+^*, \; g(kz,ky)=g(z,y)$
[G3] $\forall (z,z') \in {\Re_+}^2, \forall y \in \Re_+^*, \; g(z,z'+y)+g(z',y)=g(z+z',y)$

We will come back to h later. The properties [G2] and [G3] are illustrated by Fig. 11. According to [G2], the value $F(p,\delta,J)$, i.e., how close p is to J, is scale invariant. Now, suppose n=2, i.e., $J=J_1 \cup J_2$. Consider the gap between J_1 and J_2. If it is of negligible size, then it has negligible impact on how close p is to J (Fig. 12a). This comes from [G1] and [G3] (assume, for now, that h is continuous). Moreover, if J_2 is of negligible size, then J_2 has negligible impact on how close p is to J (Fig. 12b). This also comes from [G1] and [G3], since [G3] implies that $\forall y \in \Re_+^*, \; g(0,y)=0$ (replace z' with 0). As mentioned in Section IV.B, we have here desirable properties, which could not be obtained using Eq. 18. The summation in Eq. 20 is a legitimate choice, consistent with the additivity of distances and segment lengths (Fig. 11b). A solution to the above system of functional equations is defined by

$$\forall\, (z,y) \in \Re_+ \times \Re_+^*, \quad g(z,y) = \ln(1 + z/y) \tag{22}$$

where ln denotes the natural logarithm having base $e \approx 2.718$. The solution is unique up to a multiplicative factor. Function g as in Eq. 22 also satisfies

[G4] $\forall (z,z') \in {\Re_+}^2, \forall y \in \Re_+^*, \; z>z' \Rightarrow g(z,y)>g(z',y)$
[G5] $\forall z \in \Re_+, \forall (y,y') \in \Re_+^{*2}, \; y>y' \Rightarrow g(z,y)<g(z,y')$
[G6] $(\forall y \in \Re_+^*, g(0,y)=0)$ and $(\forall z \in \Re_+^*, \lim_{y \to 0+} g(z,y) = +\infty)$

Its range is $\Re_+$. The role of h in Eq. 20 is to guarantee that $F(p,\delta,J)$ falls into [0,1], and to allow flexibility in the shaping of the F-templates. Function h should be chosen depending on the application in hand, such that

[H1] h is a continuous mapping from $\Re_+$ into [0,1].
[H2] $\forall (x,x') \in {\Re_+}^2, \; x>x' \Rightarrow h(x) \geq h(x')$
[H3] $h(0)=0$ and $\lim_{x \to +\infty} h(x) = 1$

Suppose n=1, i.e., J is a segment. According to [G4] and [H2], the longer J, the greater $F(p,\delta,J)$, i.e., the more p is close to J. According to [G5] and [H2], the higher the distance between p and J, the less p is close to J. Once again, these are highly desirable properties. In the rest of the paper,

$$\forall\, x \in \Re_+^*, \; h(x) = \max\{0, \min(\frac{1}{k_2 - k_1}[k_2 + \frac{1}{1-e^x}], 1)\} \tag{23}$$

with $0 \leq k_1 < k_2$. When J is a segment, $F(p,\delta,J)$ varies in a very simple and predictable way. This is easy to understand: replace x with $\ln(1 + z/y)$ in Eq. 23. If the distance y between p and J is less than k_1 times the length z of J, then p is definitely close to J. If it is more than k_2 times the length of J, then p is not close to J at all. See Fig. 13. Finally, note the following conventions, consistent with [G6] and [H3], for the evaluation of $F(p,\delta,J)$ in Eq. 20:

$$\Sigma_{i \in 1..0}\, g(z_i,y_i) = 0 \;\text{ and }\; g(0,0)=0$$
$$\text{and } (\forall z \in \Re_+^*, \; g(z,0) = +\infty) \;\text{ and }\; h(+\infty) = 1 \tag{24}$$

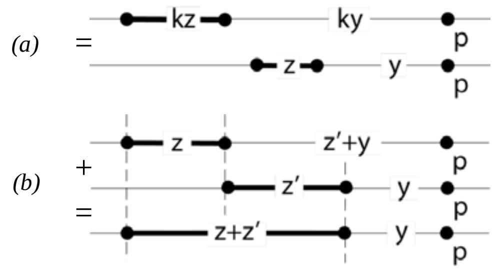

Fig. 11. ***(a)*** **Property [G2]. Scale invariance** ***(b)*** **Property [G3]. Additivity.**

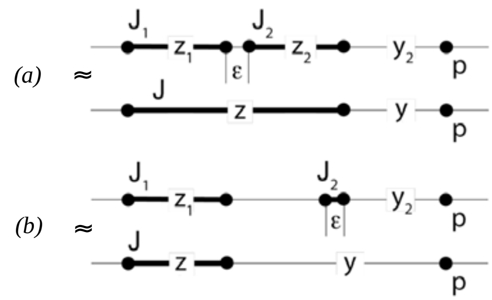

Fig. 12. ***(a)*** **A gap of negligible size has negligible impact.** ***(b)*** **A segment of negligible size has negligible impact.**

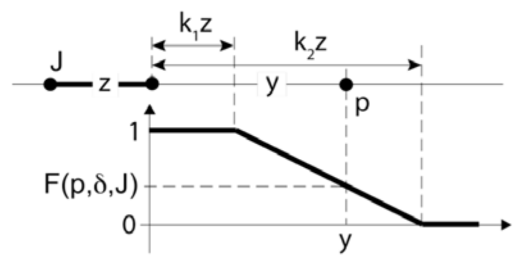

Fig. 13. When J is a segment, $F(p,\delta,J)$ varies in a very simple and predictable way.

D. *Case of fuzzy objects*

In Eqs. 20 and 21, the symbol J denotes a crisp longitudinal section. Here, we show that fuzzy sections can be handled as well. First, notice that Eq. 22 can be rewritten as

$$\forall\, (z,y) \in \Re_+ \times \Re_+^*, \quad g(z,y) = \int_y^{y+z} \frac{1}{v}\, dv \tag{25}$$

Consider a direction δ, a crisp longitudinal section J, and a point p aligned with J. Assume J is included in $\Delta^-_{J,p}$ (resp. $\Delta^+_{J,p}$). For any $v \in \Re_+$, let q_v be the point of $\Delta^-_{J,p}$ (resp. $\Delta^+_{J,p}$) whose distance to p is v. Finally, let $J(q_v) \in \{0,1\}$ be the membership degree of q_v in J. The equality in Eq. 20 can now be rewritten as

$$F(p,\delta,J) = h\left(\int_0^{+\infty} \frac{J(q_v)}{v}\, dv\right) \tag{26}$$

At this point, the extension to fuzzy objects is straightforward. Equations 20 and 21 are replaced with the following definition for $F(p,\delta,J)$. There are, again, two cases. Assume the support supp(J) of the fuzzy longitudinal section J is included in $\Delta^{-}_{J,p}$ (resp. $\Delta^{+}_{J,p}$). Let $J(q_v)\in[0,1]$ be the membership degree of q_v in J. We have

$$F(p,\delta,J) = h\left(\int_0^{+\infty} \frac{J(q_v)}{v}\, dv \right) \tag{27}$$

Now, assume $\mathrm{supp}(J) \not\subset \Delta^{-}_{J,p}$ and $\mathrm{supp}(J) \not\subset \Delta^{+}_{J,p}$. As expressed by Eq. 28, this case comes down to the previous one.

$$F(p,\delta,J) = \max\ \{\ F(p,\delta,J\cap\Delta^{-}_{J,p})\ ,\ F(p,\delta,J\cap\Delta^{+}_{J,p})\ \} \tag{28}$$

E. Examples

In this section, F_{ANG} denotes the function defined by Eq. 11, with t=min and μ as in Fig. 1a. The function F_{DIST} is defined by Eqs. 27 and 28, with h as in Eq. 23. Finally, the function F is $\min(F_{ANG}, F_{DIST})$, i.e., it is defined by Eq. 14, with t=min. The F-template $F^{\delta B}$ is, therefore, a directional F-template that embeds distance information. It is computed here for various objects B, directions δ, and parameters k_1 and k_2 (Eq. 23, Fig. 13). The role of k_1 and k_2 is illustrated by Fig. 14. The higher k_1 or k_2, the less distance information has an impact on the F-template nearby the reference object. $F^{\delta B}$ does not embed any distance information and coincides with $F^{\delta B}_{ANG}$ when $k_2=+\infty$. The two properties we wanted the F-templates $F^{\delta B}$ to satisfy are illustrated by Fig. 15. They stem from the axiomatic properties [G2] and [G3] (Section IV.C). If the reference object is scaled, the F-template is scaled by the same factor (Fig. 15a). Consider a region of negligible size relative to the size of the reference object. Whether it is part or not of the object does not have much impact on the F-template (Fig. 15b). As shown by Fig. 16b, this second property is not satisfied by $\min(F^{\delta B}_{ANG}, F^{\delta B}_{DIST})$. Remember (Section IV.A) that $\min(F^{\delta B}_{ANG}, F^{\delta B}_{DIST})$ is not an F-template. Distance information is not really embedded into the model. Rather, $\min(F^{\delta B}_{ANG}, F^{\delta B}_{DIST})$ should be seen as the result of a fusion of information from two independent sources. Finally, note that the two properties above have the following implication. If John is close to the car when 3 feet from the driver's door, then Mary is even closer to the car when 3 feet from the front bumper. (Unless the vehicle is as wide as it is long.) This is well illustrated by Fig. 16a. In the F-template, regions of high membership values tend to stretch more along the major axes of the reference object (which is here quite elongated).

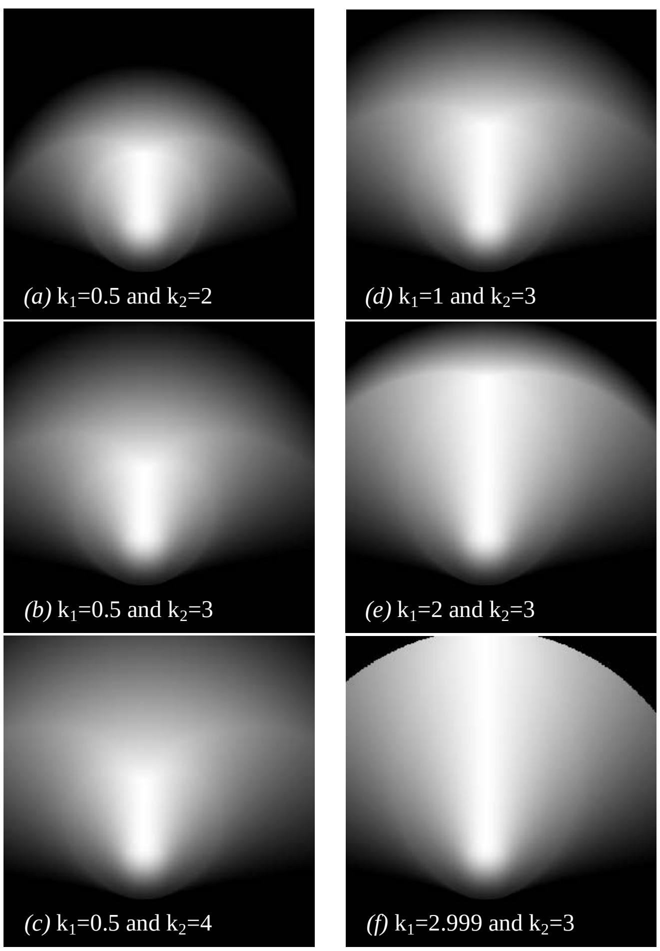

Fig. 14. Shaping the F-templates: the role of k_1 and k_2. The reference object is as in Fig. 8a. *(a)(b)(c)* Constant k_1, increasing k_2. *(d)(e)(f)* Increasing k_1, constant k_2.

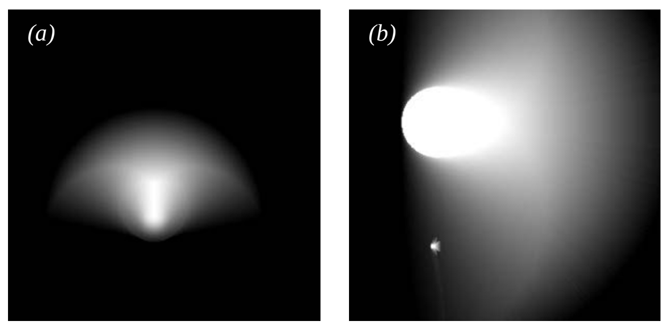

Fig. 15. Fundamental properties. Here, $k_1=0.5$ and $k_2=3$. *(a)* Scale invariance. The reference object is as in Fig. 8a, but half its size. Compare with Fig. 14b. *(b)* A region of negligible size has negligible impact. The reference object is as in Fig. 10a.

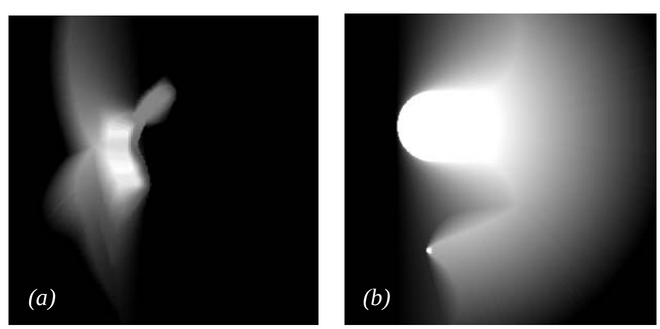

Fig. 16. A last example and a counterexample. Here, $k_1=0.5$ and $k_2=3$. *(a)* F-template induced by the object as in Fig. 6a. *(b)* The fuzzy set $\min\{F^{\delta B}_{ANG}, F^{\delta B}_{DIST}\}$. It is not an F-template. The reference object is as in Fig. 10a.

V. Conclusions and Future Work

A directional spatial relationship to a reference object can be represented by a spatial template, i.e., a fuzzy subset of the Euclidean space. These templates (which are given different names in the literature) play an important role in object localization tasks. In previous work, we modeled them through *F-templates* and discussed the case of crisp 2D objects in raster form. We introduced a new algorithm for directional spatial template computation, which is faster, gives better results and is more flexible than its competitors.

Here, we have shown that our algorithm can easily be extended to handle fuzzy objects, while keeping its advantages. In existing models, only angular deviation is taken into account. The issue has been examined, and directional F-templates embedding distance information have been designed to elegantly satisfy two interesting properties. The shape of the F-templates, i.e., the impact of distance information, can be controlled with great flexibility using two very intuitive parameters.

In future work, we intend to submit our model to extensive experiments, and further explore the issue above. The concept of the F-template is dual to that of the F-histogram, which can handle not only 2D objects and raster data, but also 3D objects and vector data [15] [14] [4] [16]. We will show that F-templates have the same capabilities.

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