Simulation Using Spreadsheets

- Objectives on basics of simulations
  - Simulating randomness
  - Simulating binary events
  - Simulating service and interarrival times
- Objectives on simulation applications
  - Simulating queuing systems
  - Simulating inventory systems
  - Simulating system maintenance
- Reference: Chapter 2

Alternative Tools for Simulation

- Alternatives to program a model
  1. Spreadsheets
  2. Simulation environments/packages
  3. Simulation programming languages
  4. General purpose programming languages
- We will use spreadsheets to illustrate key concepts and key components of simulation in this unit.
- We will implement some simulations with Java later.

Simulation of Random Numbers

- A stochastic model must simulate randomness.
  - Ex Call center interarrival times and service times
- A random number is a randomly generated real number in [0,1].
- A random number generator (RNG) is a method for generating a sequence of random numbers.
  - Ex Excel built-in RNG: `RAND()`
  - Ex User-defined RNG in VBA: `Rnd()` and `Rnd01()`

Simulating Binary Events Using Random Numbers

- Binary event: An event of 2 possible outcomes
  - Ex Coin tossing
  - Ex Listening to an Ethernet cable
- Ex Simulating coin toss
- How to simulate binary events
- What type of simulation model is it?
Properties of Random Number Generators

- A RNG should satisfy some desirable properties.
  1. Numbers should be uniformly distributed.
  2. They should be statistically independent.
  3. The sequence should be reproducible.

Limitations of RAND() and Rnd()

- Short period
- Uniformity and independence
- Reproducibility
- Not suitable for professional work

Uniformity and Independence

- Uniformity: Probability that a number falls in an interval \((a, b) \subseteq [0,1]\) equals the length \(b-a\).
- How do we test uniformity of a given sequence?
  - Ex A sequence of generated numbers
    \[0.11,0.85,0.32,0.64,0.25,0.76,0.47,0.58,0.09,0.93, \ldots\]
- Independence: Knowing previously generated numbers cannot help predicting the next number.

RNGs from General Purpose Languages

- Java Random class
  ```java
  Random rng = new Random( 19580427 );
  double r = rng.nextDouble();
  ```
  - Range \([0,1]\)
  - Period and reproducibility

Simulating binary events

- Denote probability \(P(out1)\) by \(p\), where \(p \in (0,1)\)
- \(out = (rng.next() < p) \iff out1 : out2;\)

Simulating Random Sample

from Discrete Distribution - Example

- Ex Service time
  - An automated phone info service spends 3, 6, or 10 minutes for each caller.
  - Portions of calls for each length of service are 30%, 45%, and 25%, respectively.
  - Generate a service time sample of size 25.
  - How does the simulation work?
- General case
  - A discrete variable \(x\) is defined with its possible values.
  - A discrete probability distribution \(P(x)\) is given.
  - Task: Generate a sample of \(x\) with a given size \(m\).
Simulating Random Samples from Discrete Distributions - Method

• Method
  1) Order values of x as \( \{x_1, x_2, ..., x_n\} \).
  2) Order probabilities consistently as \( \{P(x_1), P(x_2), ..., P(x_n)\} \).
  3) Derive cumulative probabilities \( \{C(x_1), C(x_2), ..., C(x_n)\} \), where
    \[
    C(x_i) = \sum_{j=1}^{i} P(x_j) .
    \]
  4) To generate a sample \( \{y_1, ..., y_m\} \), simulate a sequence of random numbers \( \{r_1, ..., r_m\} \).
  5) If \( C(x_{k-1}) \leq r_i < C(x_k) \), then \( y_i = x_k \) (\( k=1, ..., n \)), where \( C(x_0) = 0 \).

• Ex \( x \in \{a, b, c\} \) with \( P(x) \) over \( \{a, b, c\} = (0.3, 0.5, 0.2) \)
  - What if we order values of x as \( \{c, a, b\} \)?

Spreadsheet Implementation

• VBA function
  - \texttt{DiscreteEmp(rangeCumProb, rangeValue)}
  - Ex Service times

• Can it be implemented without using \texttt{DiscreteEmp()}?

• Is the following implementation correct?
  - \texttt{IF(RAND()<=0.3, 3, IF(RAND()<=0.75, 6,10))}

Simulating Random Arrival Times

• Ex Arrival times
  - Calls to an automated phone info service have interarrival times 1, 2, 3, or 4 minutes, with equal probability.
    - Generate a sample of call arrival times of size 26 with the initial call arriving at \( t_0 = 0 \).

• General case
  - Given a discrete distribution of interarrival time \( A \) and arrival time \( t_0 \) of initial call, simulate a sample of \( m \) arrival times.

• Method
  1) Simulate a sample of \( m-1 \) interarrival times \( \{A_1, A_2, ..., A_{m-1}\} \).
  2) From \( i=1 \) to \( m-1 \), get arrival time \( t_i = t_{i-1} + A_i \).
  3) The sample of arrival times is \( \{t_0, t_1, ..., t_{m-1}\} \).

Activity Time and Event Time

• An activity occurs over a duration of time.
  - Activity time measures the length of the duration.
  - Ex Service time

• An event occurs instantly.
  - Event time is the clock time when the event occurs.
  - Ex Arrival time
    - Clock in a simulation starts from \( t_0 \) which may or may not be equal to 0.
Framework for Spreadsheet Simulation

- Components
  - Input, state, output, simulation table, and response
- Model input
  - Exogenous parameters from the environment
- System state
  - Variables describing the current status of the system
- Model output
  - Measures of aspects of the system used to compute model response below.
- Model response
  - Measures of system performance

Simulation Table

- Each column is one of the following.
  - An event or event time
  - An activity time
  - A system state variable
  - A model output measure
- Each row represents a simulation step.
  - Associated with occurrence of events
  - Depend solely on model input and previous steps

Guideline for Spreadsheet Simulation

1. Determine model input parameters
2. Determine relevant activities, events, and system states
3. Determine necessary model response measures
4. Determine model output measures needed to compute model response
5. Formulate simulation table column headings
6. Populate simulation table row by row
7. Compute model response from model output

Queuing Systems

- Business, industry, and engineering operations often involve waiting lines for services. How to best setup and manage them are important management decisions.
  - Ex PC technical support call center
  - Ex Assembly lines in manufacturing
- A queuing system involves a calling population of units, one or more waiting lines, and one or more servers.
- Arrival rate and distribution of interarrival time
- Types of queuing systems to focus on
  - Unlimited calling population
  - Constant arrival rate
  - Unlimited system capacity
  - FIFO
  - Arrival rate < service rate
State and Events of Queuing Systems

- **System state**
  - Q: Number of units in queue
  - U: Number of units in the system
  - Server status
  - Relation between units in system and server status

- **Relevant events**
  - Arrival event
  - Service start event
  - Departure event

How Events Affect State

- **How does the system state change when an arrival event occurs?**
  - Flow diagram for arrival
  - Changes of state on arrival

- **How does the system state change when a departure event occurs?**
  - Flow diagram for departure
  - Changes of state on departure

Simulating Single-Server Queues

- **Ex** A sample of 6 units in a single-channel queue
  - Customer IDs: (1, 2, 3, 4, 5, 6); initial arrival at t = 0
  - Interarrival times: (2, 4, 1, 2, 6)
  - Service times: (2, 1, 3, 2, 1, 4)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer ID</td>
<td>Interarrival Time</td>
<td>Arrival Time</td>
<td>Service Start Time</td>
<td>Service Time</td>
<td>Service End Time</td>
</tr>
</tbody>
</table>

- **Use event diagram to determine columns D and F.**
- **What is the numerical formula for Service Start Time?**
- **What about Service End Time?**

Computing Model Output

- **Ex** Single server: A corner store has one checkout counter. Interarrival times range from 1 to 8 minutes with equal probability. Service times vary from 1 to 6 minutes.

- **Simulating interarrival times of equal probability**
  - The simulation table
  - VBA function: `DiscreteUniform(rangeLow, rangeHigh)`

- **Calculating model output**
  1) The time waiting in queue by unit i
  2) The time spend in system by unit i
  3) Server idle time btw departure of unit i-1 and that of unit i
Computing Model Response

1. Average waiting time per unit
2. Estimate of probability that a unit has to wait
3. Percentage server idle time
4. Average service time per unit
5. Average interarrival time
6. Average waiting time per waiting unit
7. Average in-system time per unit

Trials and Experiments

- **A trial** is the simulation of one sample of a particular size.
  - A model response is derived from a trial.
  - Due to randomness, model response varies from trial to trial.
  - Each trial simulates a particular day.
- **An experiment** involves multiple trials.
  - Experiments can be used to study model response across multiple trials.
  - Can also be used to study how model response varies between trials.

Simulating Two-Server Queues

- Ex Able and Baker
  - A call center is served by Able and Baker.
  - Each server has a different service time distribution.
  - When both servers are idle, Able takes the next call.
- Key issue
  - For each call, which server will serve it?

Who Serves The Arrival Call?

- For each arrival call $C_i$, define $T_a(S)$ as the earliest time when server $S$ is available.
  - For the 1st call, $T_a(S) = t_0$.
  - For other calls, $T_a(S) =$ completion time of the last call $S$ served.
  - How can $T_a(S)$ be calculated?
- Once $T_a(S)$ is known, who serves call $C_i$ can be resolved.
  - if ( $T_a($Able$) \leq$ arrival time of $C_i$ )
    - $T_a($Able$) \leq T_a($Baker$)$ server = Able;
    - else server = Baker;
  - Ex Arrival time (0,1,2,3) and service time (3,5,3,4)
Other Event Times and Model Response

- **Service start time**
  - Determine based on chosen server, arrival time and server available time.
- **Model output**
  - Caller delay
  - Caller system time
- **Model response**
  - Average caller delay
  - Max caller delay

Inventory System Simulation

- **Inventory or stock** refers to goods and materials that a business holds for the purposes of resale (possibly after value-added manufacturing).
  - Example: Treated lumbers in a Home Depot store
  - Example: Foods to be canned in a canned food manufacture
- The primary objective of an inventory system is to control stock levels in order to balance availability of product against stock holding cost.

Simulating Single-Period Inventory

- **Ex News dealer**
  - A news dealer buys papers for $0.33 each, sells for $0.50 each, and sells outdated ones for $0.05 each.
  - Papers are purchased in bundles of 10.
  - A newsday (NT) may be good, fair or poor.
  - Daily demand (De) is in bundles of 10 btw 40 and 100.
  - Each type of newsday has a different demand distribution.
    - P(De|NT) is a conditional probability distribution.
  - Daily profit = PaperSale + ScrapSale – PaperCost – LostProfitFromExcessDemand
  - What is the 20-day profit with daily order of 70 papers?

Simulation Table and Model Response

- **Simulate type of newsday (NT)**
- **Simulate daily demand (De)**
  - Embed service-time-like simulations in IF statement
- **Determine paper sale revenue**
- **Determine scrap sale revenue**
- **Compute LostProfitFromExcessDemand**
- **Output**: daily profit
- **Response**: total profit over 20 days
  - Trade-off between large and small daily purchase
(M,N) Inventory Systems

- **M**: maximum inventory level
- **N**: Inventory is reviewed every N time units, and an order is made to bring inventory up to level M.
- **Lead time** is the length of time period between placing an order and receiving it.
  - May not be constant
- **Shortage** is the amount of demand arrived after inventory drops to zero.
  - Two possible outcomes
- Cost: items, carrying, ordering, lost sales, and scrap
- Model response: typically total profit

Simulating (M,N) Inventory

- **Ex Refrigerator Inventory**
  - A refrigerator store reviews inventory every N = 5 days and places an order to replenish to M = 11 fridges.
  - Order = M – EndingInventory + Shortage
  - Daily demand is between 0 to 4 fridges.
  - Lead time is between 1 to 3 days.
    - Interpretation
- **Task**: Simulate a sample of 25 days.
  - Initial inventory = 3, previous order = 8, lead time = 2.
  - A trace of initial days

Simulation Table: Clock and Order

- **Clock time related**
  - Use day 0 to accommodate initial condition.
  - Organize days in each review period into a cycle.
  - Index days within a cycle from 1 to 5.
- **Order related**
  - Order is placed on the 5th day of each cycle.
  - Lead time is simulated on the 5th day of each cycle.
  - “Days until order arrives” indicates time for inventory update.

Inventory and Model Response

- **Inventory related**
  - Daily beginning inventory depends on the ending inventory from yesterday and the arrival order.
  - Shortage is updated daily.
  - Daily ending inventory
- **Model response**
  - Daily ending inventory is both a state variable and a model output.
  - It’s a useful indicator to the efficiency of inventory policy.
System Maintenance Policy

- Machines are widely used in industry, business, and daily life.
- All machines break down due to wear and tear.
- When machines break down, they bring costs of part replacement, downtime, and repair time of maintenance staff.
- Proper system maintenance policy can reduce the total cost due to breaking down.

Simulating System Maintenance Policy

- Ex Bearing replacement
  a. A milling machine has 3 bearings, each with a life span between 1000 to 1900 hours.
  b. When a bearing fails, a new one costs $32.
  c. Machine downtime due to failure costs $10 per minutes.
  d. Upon failure, a mechanic arrives between 5 to 15 minutes.
  e. It takes 20, 30, or 40 min to change 1, 2, or 3 bearings.
  f. On-site cost of mechanic is $30 per hour.
- Task: Simulate a sample of 15 changes per bearing
  - Performance measure: total cost per 10,000 bearing-hours

Simulation

- Simulation table
  - Assumption: Bearings do not fail at the same time.
  - Each bearing can be simulated independently.
  - Each step simulates 3 pairs of life span and arrival time.
  - Repair time is constant.
- Cost breakdown
  1) Cost of bearing
  2) Cost of mechanic delay
  3) Cost of repair downtime
  4) Cost of mechanic
- Total cost per 10,000 bearing-hours

Ex Evaluating Alternative Policy

- Alternative policy: Replace all 3 bearings if one fails.
- Simulation table
  - Life span of first failing bearing is the effective life of all bearings.
  - Only one set of mechanic delay times needs simulated.
- Cost evaluation
  1) Cost of bearing
  2) Cost of mechanic delay
  3) Cost of repair downtime
  4) Cost of mechanic
  5) Cumulative life
  6) Total cost per 10,000 bearing-hours