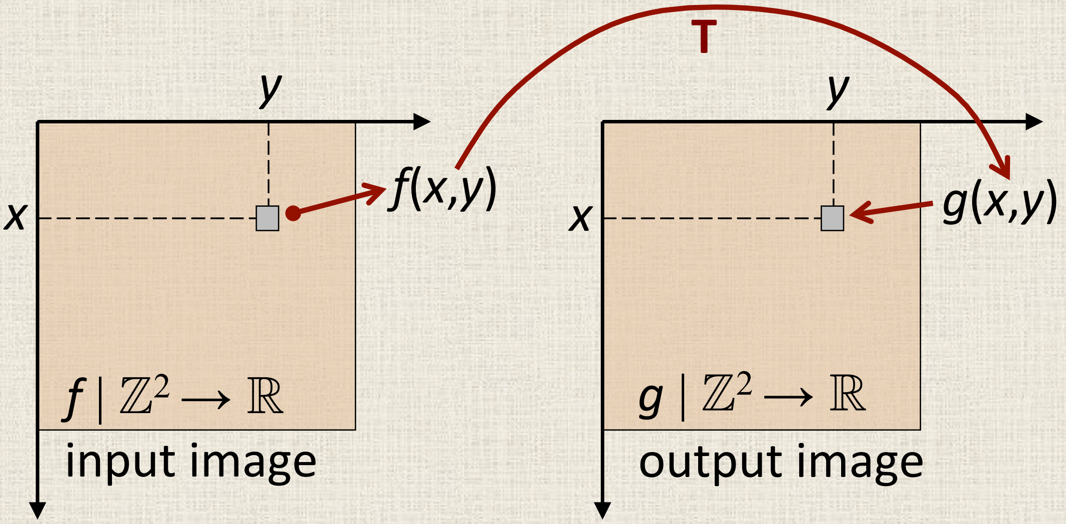
B

Image Operations and Transformations

Image Operations and Transformations

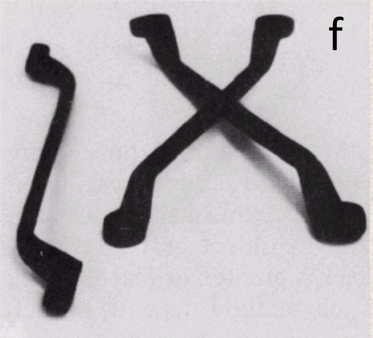
I. Single-Pixel Operations

I.1. Principle



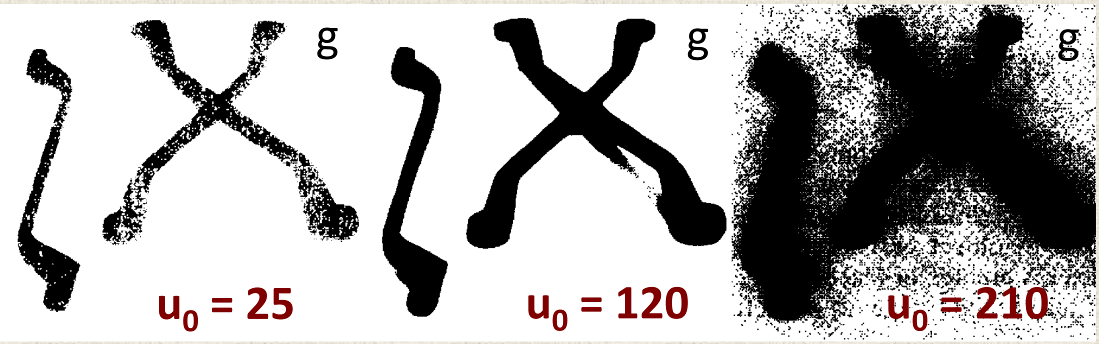
$$g(x,y) = \mathrm{T}(f(x,y))$$

I.2. Thresholding



8-bit grayscale image

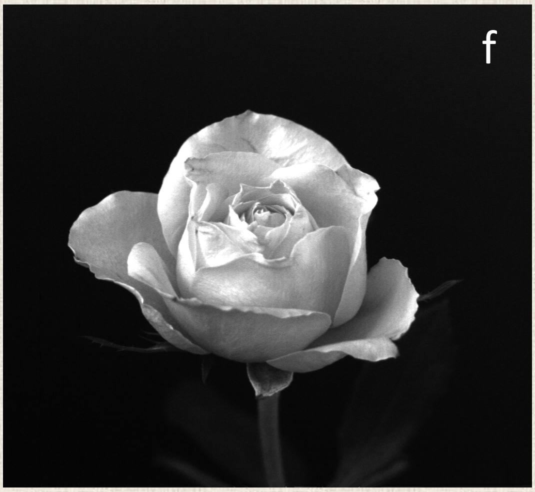
If $u \le u_0$ then $T(u)=0$
If $u > u_0$ then $T(u)=255$



$u_0 = 25$ $u_0 = 120$ $u_0 = 210$

I.3a. Linear Gray-Level Mapping

$$\mathrm{T} \left| \begin{array}{l} \mathbb{R} \to \mathbb{R} \\ \mathrm{u} \mapsto a\mathrm{u}+b \end{array} \right.$$

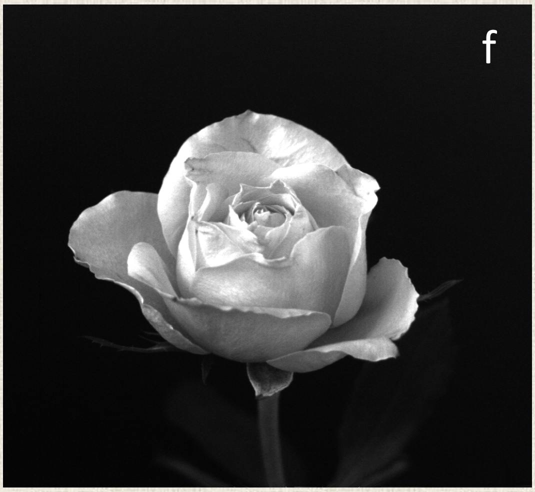


8-bit grayscale image

***a*=? *b*=?**

I.3b. Linear Gray-Level Mapping

$$\mathrm{T} \left| \begin{array}{l} \mathbb{R} \to \mathbb{R} \\ \mathrm{u} \mapsto a\mathrm{u}+b \end{array} \right.$$



8-bit grayscale image

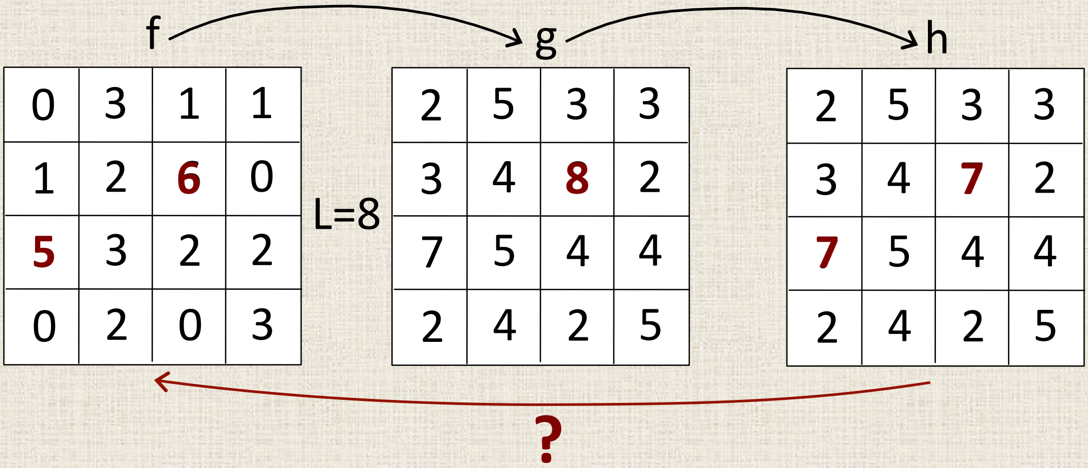
***a*=? *b*=?**

I.4a. Issue

Assume: $\exists(x,y)\in\mathbb{Z}^2$, $g(x,y)\notin 0..L\text{-}1$

"Viewable" version, h, of g? Here is a solution:

$\forall(x,y)\in\mathbb{Z}^2$, $h(x,y)=\text{nint}[\min\{L\text{-}1,\max\{0,g(x,y)\}\}]$



f

0	3	1	1
1	2	**6**	0
5	3	2	2
0	2	0	3

g

2	5	3	3
3	4	**8**	2
7	5	4	4
2	4	2	5

h

2	5	3	3
3	4	**7**	2
7	5	4	4
2	4	2	5

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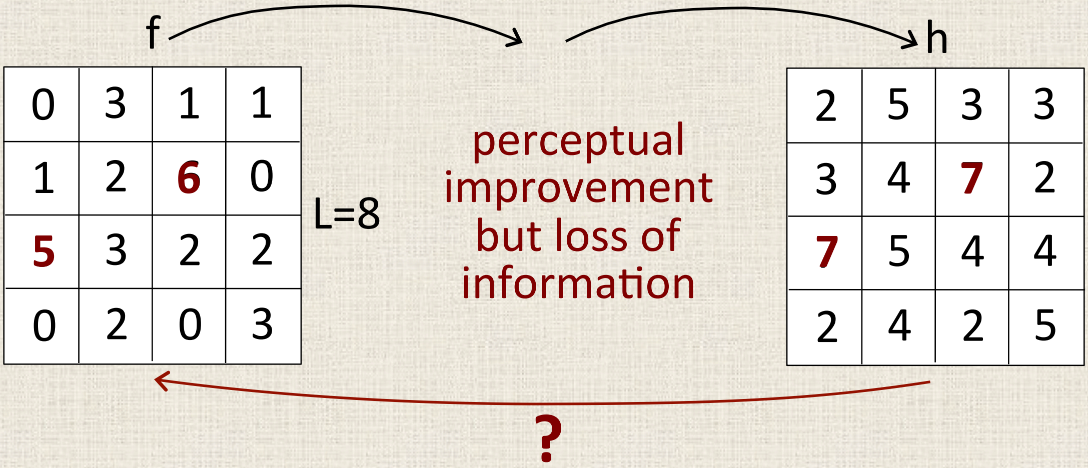
I.4b. Issue

Assume: $\exists(x,y)\in\mathbb{Z}^2$, $g(x,y)\notin 0..L\text{-}1$

"Viewable" version, h, of g? Here is a solution:

$\forall(x,y)\in\mathbb{Z}^2$, $h(x,y)=\text{nint}[\min\{L\text{-}1,\max\{0,g(x,y)\}\}]$



perceptual improvement but loss of information

f

0	3	1	1
1	2	**6**	0
5	3	2	2
0	2	0	3

h

2	5	3	3
3	4	**7**	2
7	5	4	4
2	4	2	5

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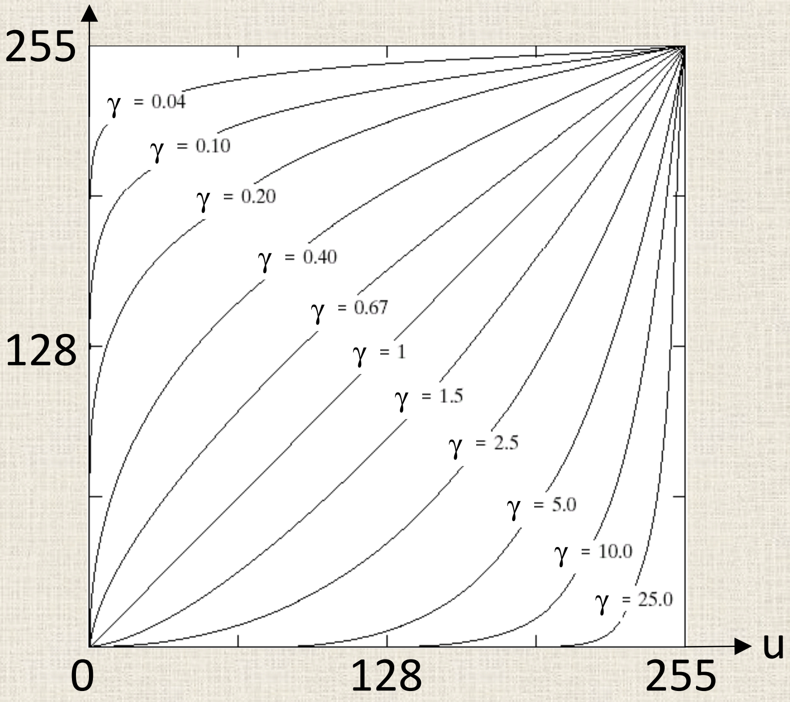
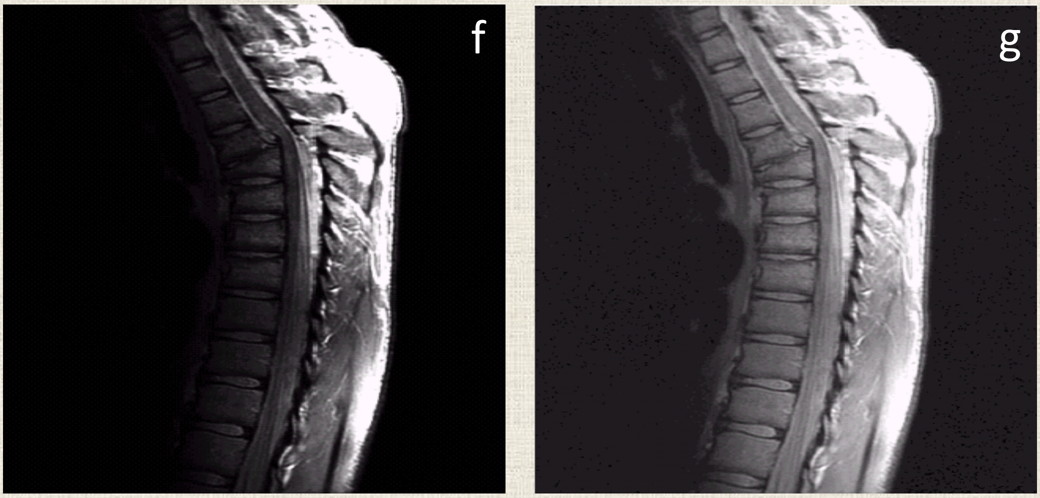
I.5a. Power-Law Gray-Level Mapping

$T(u)=255(u/255)^{\gamma}$



I.5b. Power-Law Gray-Level Mapping

$T(u)=255(u/255)^{\gamma}$



γ = ?

I.5c. Power-Law Gray-Level Mapping

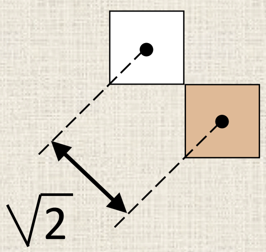
$$T(u)=255(u/255)^{\gamma}$$

γ = ?

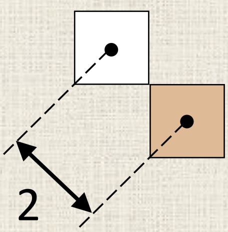
Image Operations and Transformations

II. Neighborhood Operations

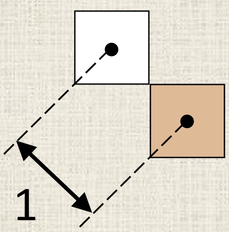
II.1. Neighborhoods



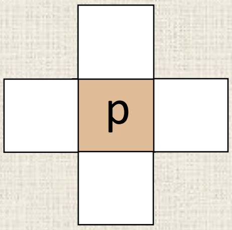
Euclidean
d_E



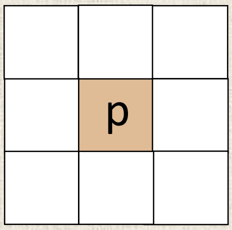
City-block
d_4



Chessboard
d_8

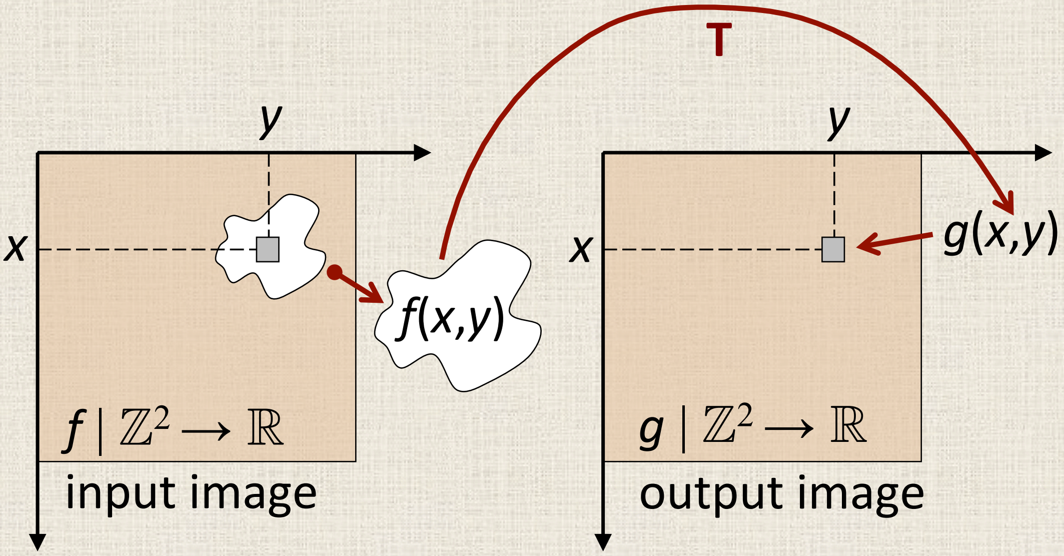


4-neighbors of p



8-neighbors of p

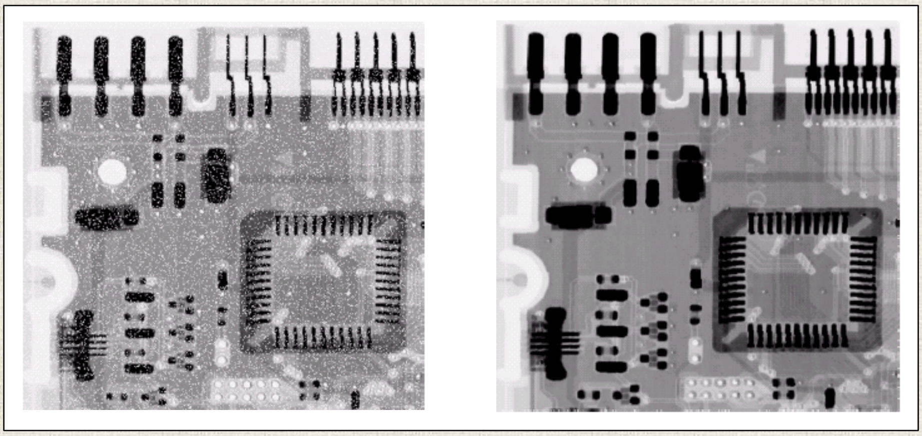
II.2. Principle



g(*x*,*y*) depends on the gray levels from some neighborhood of (*x*,*y*) in *f*.

II.3a. Examples: Min Filters

X-ray image of a circuit board corrupted by salt noise



3x3 ***min filtering***

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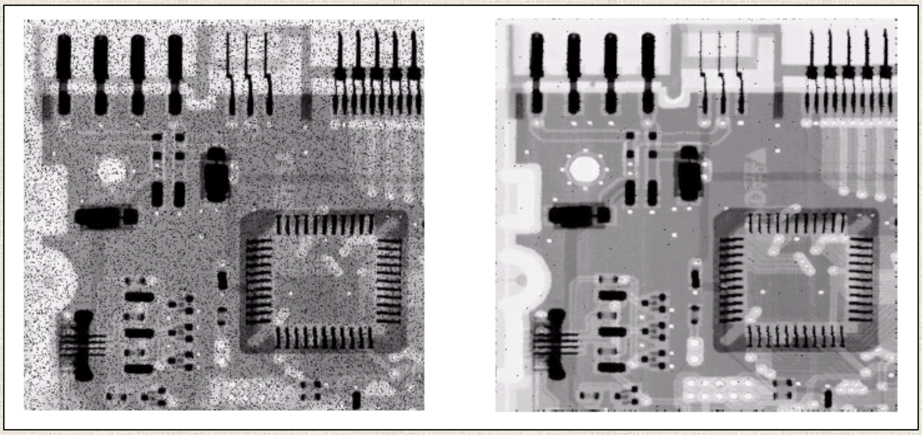
II.3b. Examples: Max Filters

X-ray image of a circuit board corrupted by pepper noise



3x3 ***max filtering***

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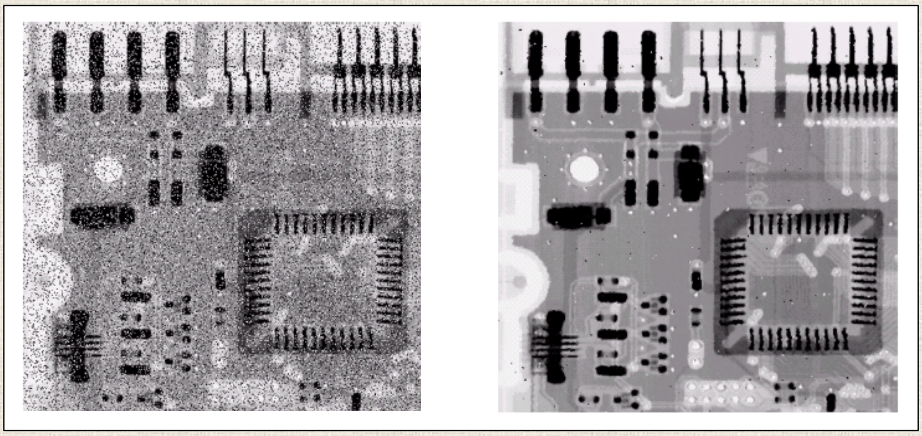
II.3c. Examples: Median Filters

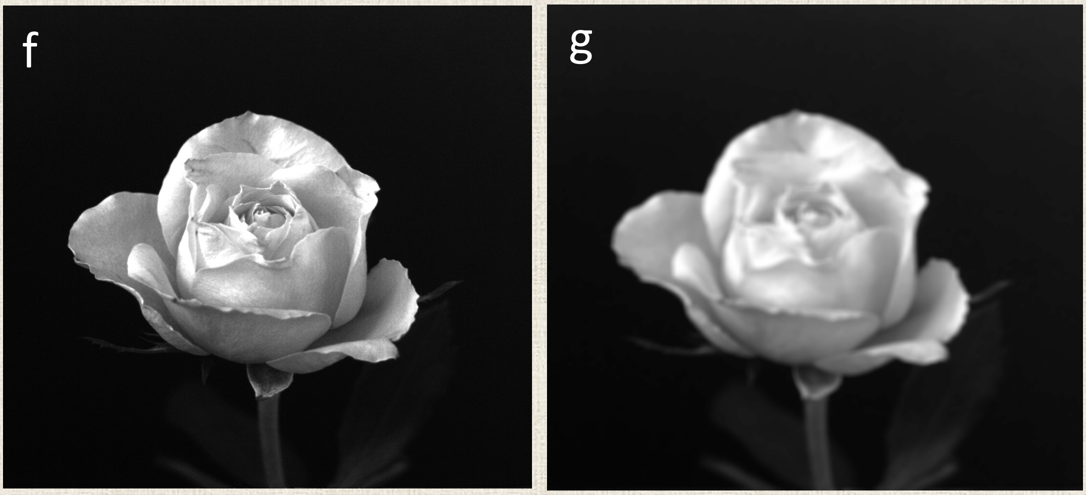
X-ray image of a circuit board corrupted by salt-and-pepper noise



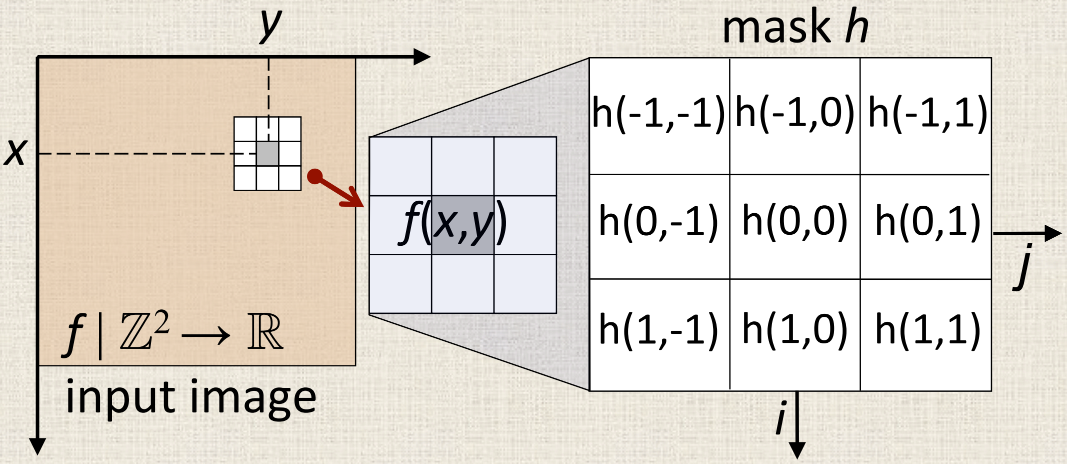
3x3 ***median filtering***

II.3d. Examples: Mean Filters

g(*x*,*y*) is the average of the gray levels from some neighborhood of (*x*,*y*) in *f*.



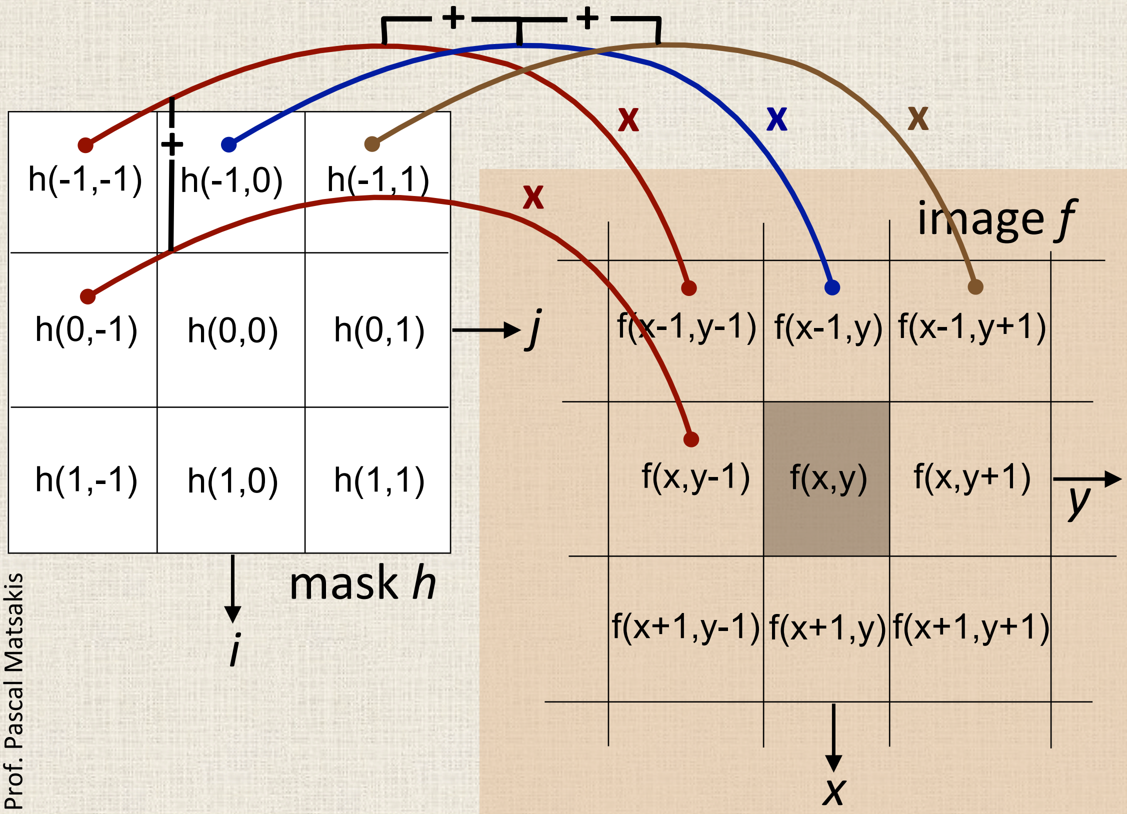
II.4a. Correlation



h(-1,-1)	h(-1,0)	h(-1,1)
h(0,-1)	h(0,0)	h(0,1)
h(1,-1)	h(1,0)	h(1,1)

The gray levels are weighted coefficients that come from a **mask** *h*.

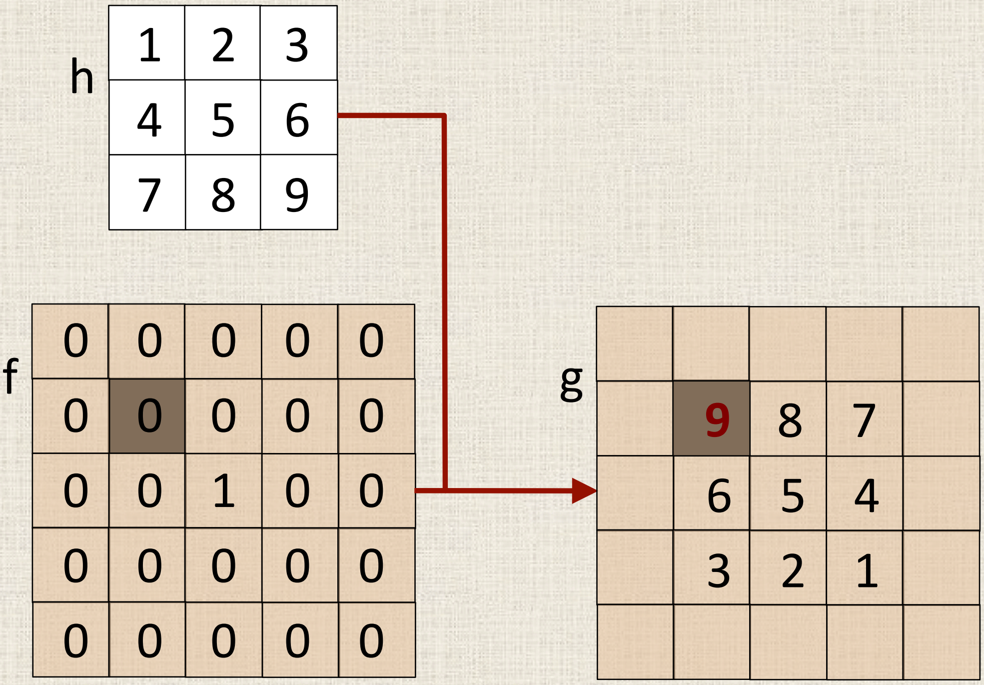
II.4b. Correlation



h(-1,-1)	h(-1,0)	h(-1,1)
h(0,-1)	h(0,0)	h(0,1)
h(1,-1)	h(1,0)	h(1,1)

f(x-1,y-1)	f(x-1,y)	f(x-1,y+1)
f(x,y-1)	f(x,y)	f(x,y+1)
f(x+1,y-1)	f(x+1,y)	f(x+1,y+1)

II.4c. Correlation



h

1	2	3
4	5	6
7	8	9

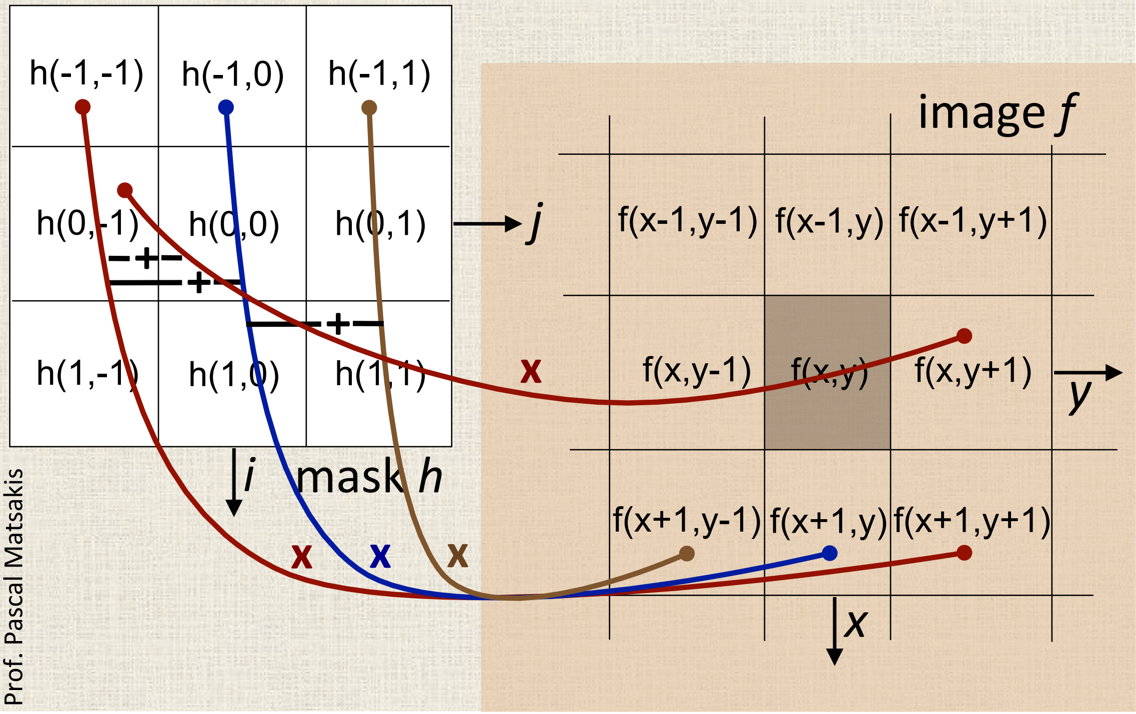
f

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

g

	9	8	7	
	6	5	4	
	3	2	1	

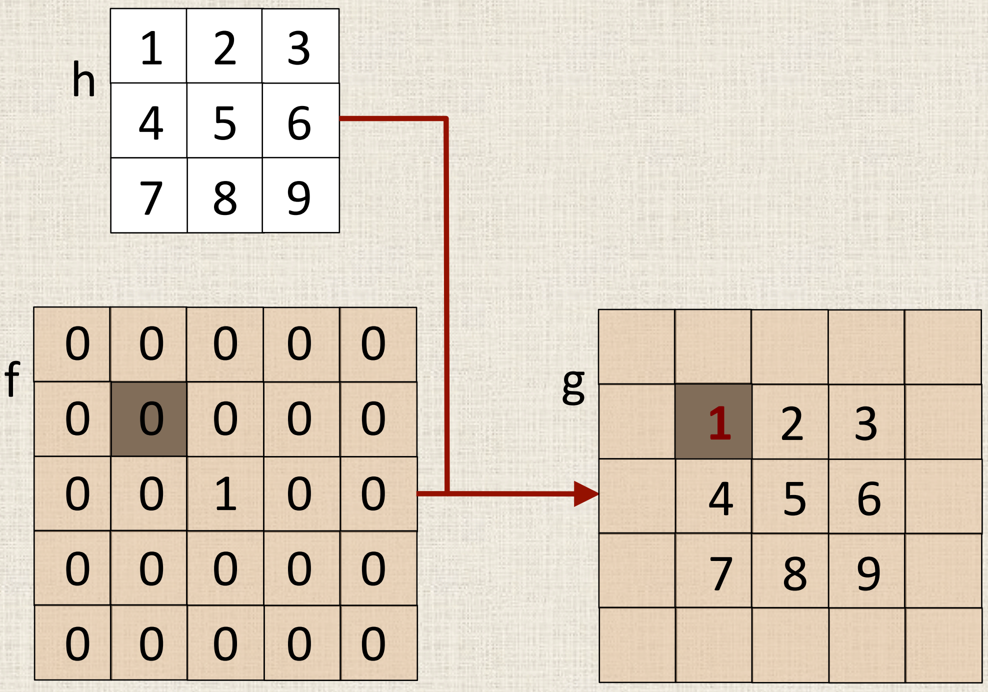
II.5a. Convolution



h(-1,-1)	h(-1,0)	h(-1,1)
h(0,-1)	h(0,0)	h(0,1)
h(1,-1)	h(1,0)	h(1,1)

f(x-1,y-1)	f(x-1,y)	f(x-1,y+1)
f(x,y-1)	f(x,y)	f(x,y+1)
f(x+1,y-1)	f(x+1,y)	f(x+1,y+1)

II.5b. Convolution



h

1	2	3
4	5	6
7	8	9

f

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

g

	1	2	3	
	4	5	6	
	7	8	9	

II.5c. Convolution

Consider an $m{\times}n$ mask h and an image f.
Consider the image g defined by:

$$\underset{\Big\downarrow \atop h*f}{g(x,y)} = \underbrace{\sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} \sum_{j=-\frac{n-1}{2}}^{\frac{n-1}{2}} h(i,j)\, f(x-i,y-j)}_{(h*f)(x,y)}$$

$h*f$ is the **convolution** of h with f
h is the **convolution kernel**

II.6a. Examples (cont'd): Mean Filters

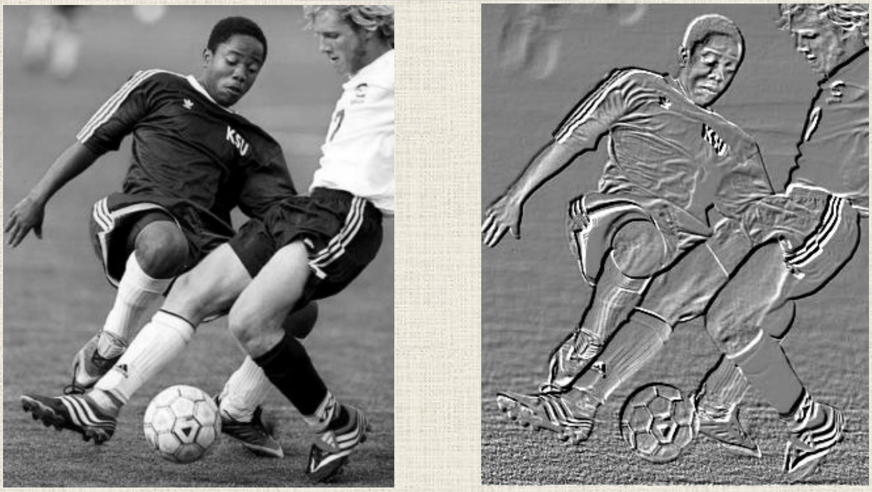
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

II.6b. Examples (cont'd): Mean Filters

1	1	1
1	1	1
1	1	1

II.6c. Examples (cont'd): Embossing Filters

2	0	0
0	-1	0
0	0	-1



II.6d. Examples (cont'd): Prewitt Kernels

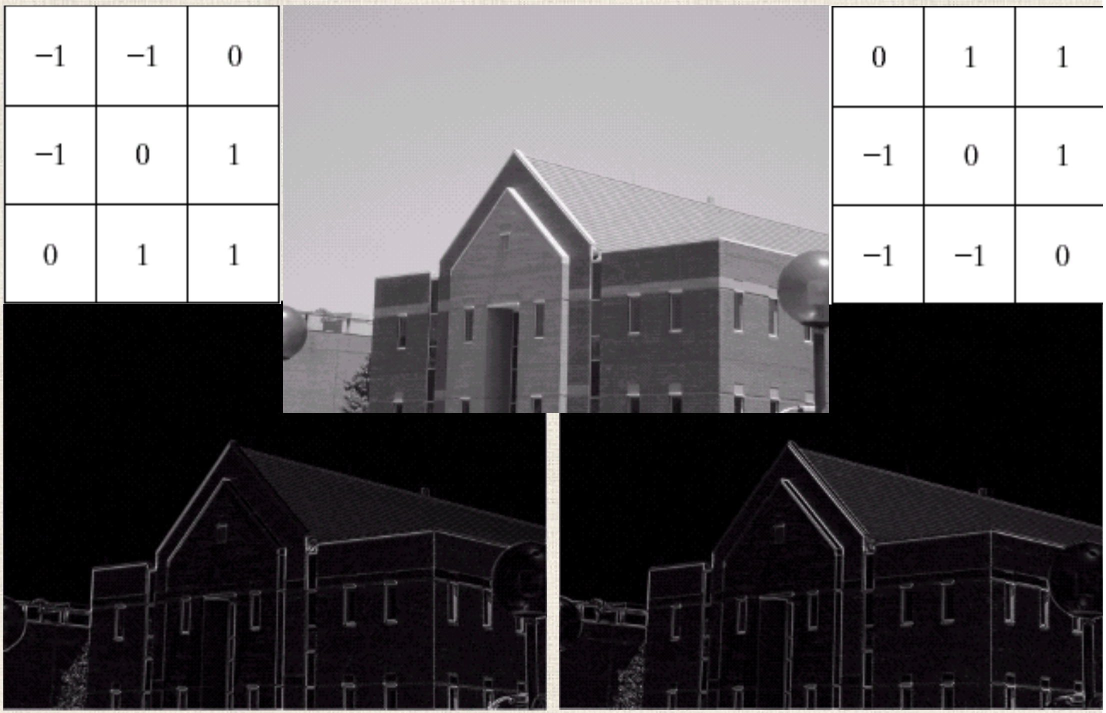
−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

horizontal edges

vertical edges

II.6e. Examples (cont'd): Prewitt Kernels



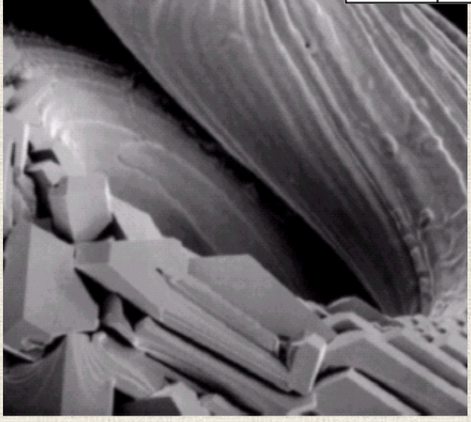
−1	−1	0
−1	0	1
0	1	1

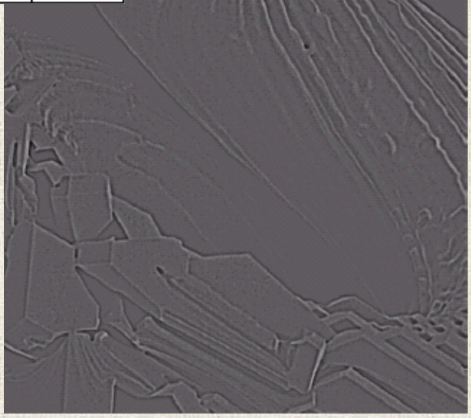
0	1	1
−1	0	1
−1	−1	0

↗ diagonal edges ↘ diagonal edges

II.6f. Examples (cont'd): Laplacian Kernels

−1	−1	−1
−1	8	−1
−1	−1	−1





II.7. Issues

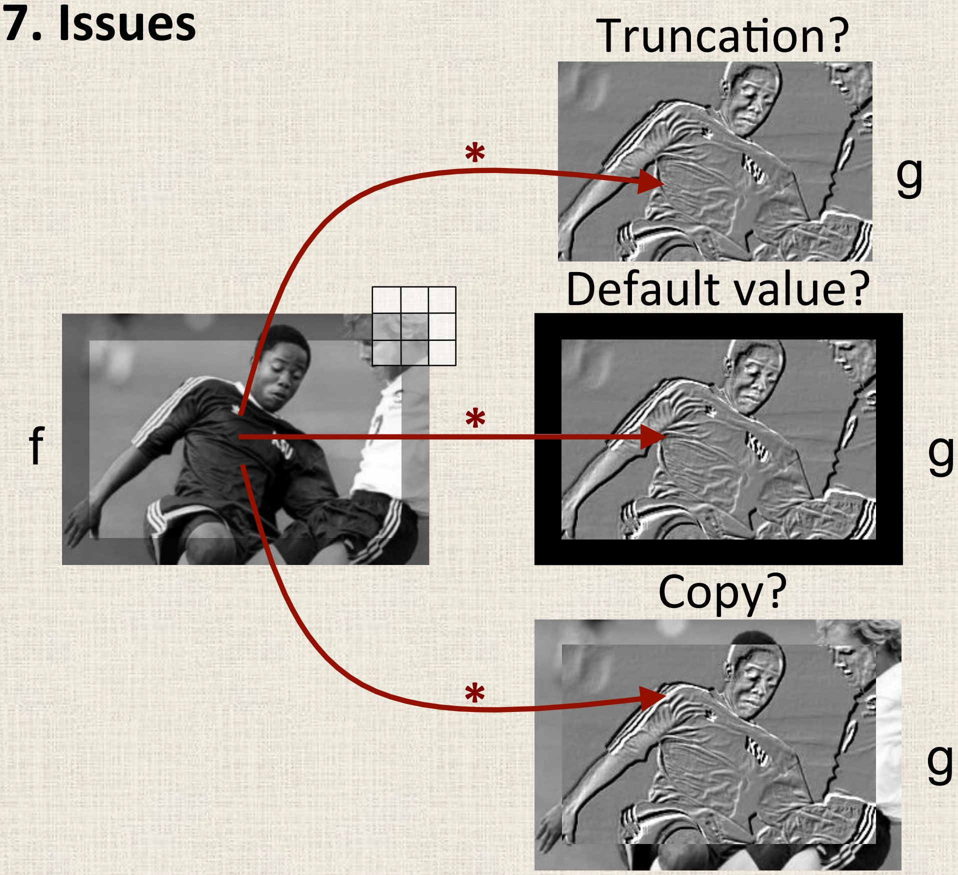
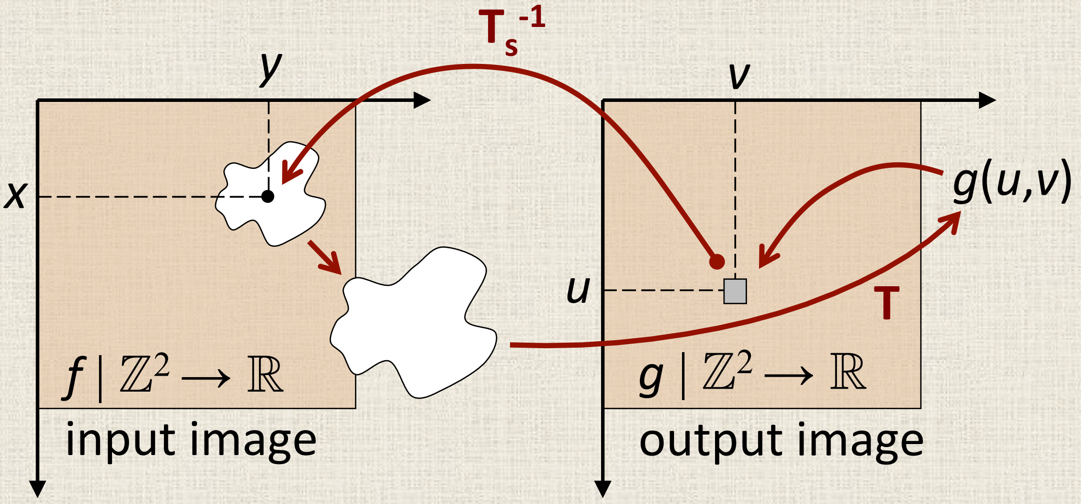


Image Operations and Transformations

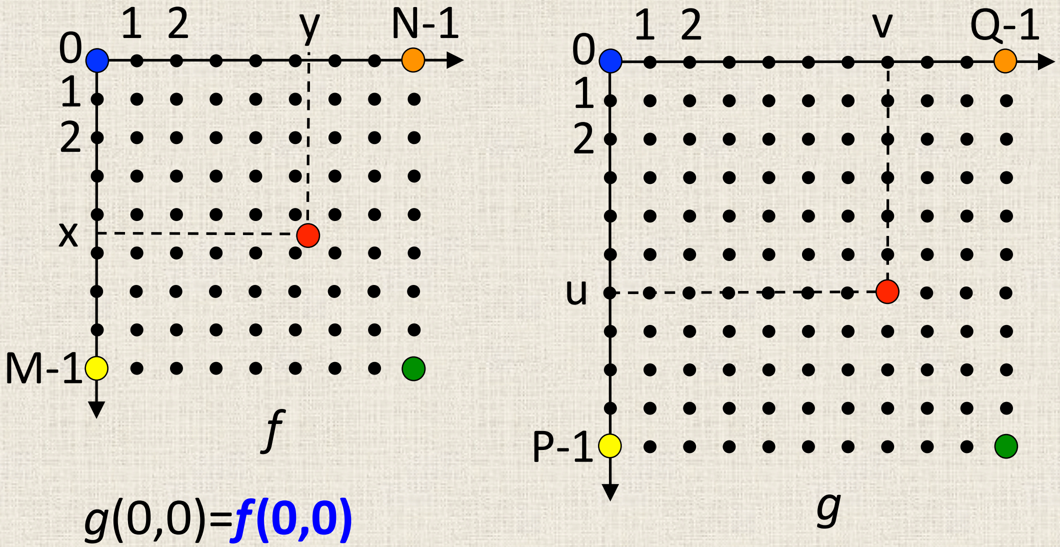
III. Geometric Spatial Transformations

III.1. Principle



g(*u*,*v*) depends on the gray levels from some neighborhood of (*x*,*y*) in *f*.

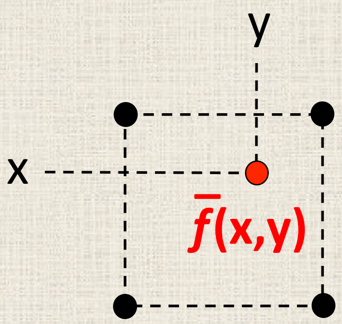
III.2a. Example: Image Scaling



$g(0,0)=f(0,0)$
$g(0,Q-1)=f(0,N-1)$
$g(P-1,0)=f(M-1,0)$
$g(P-1,Q-1)=f(M-1,N-1)$

$g(u,v)=\bar{f}(x,y)$
with x=? and y=?

III.2b. Example: Image Scaling



Nearest neighbour:

$\bar{f}(x,y)=f(X,Y)$

with X=? and Y=?

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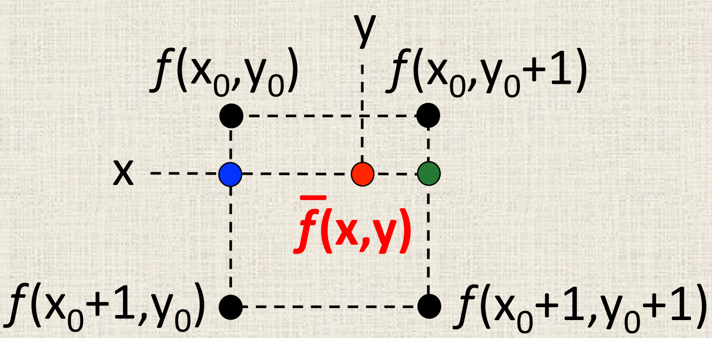
III.2c. Example: Image Scaling



Bilinear interpolation:

$\bar{f}(x,y_0)=(1\text{-}s).f(x_0,y_0)+s.f(x_0+1,y_0)$

$\bar{f}(x,y_0+1)=(1\text{-}s).f(x_0,y_0+1)+s.f(x_0+1,y_0+1)$

$\bar{f}(x,y) = (1\text{-}t).\bar{f}(x,y_0)+t.\bar{f}(x,y_0+1)$

with s=? and t=?

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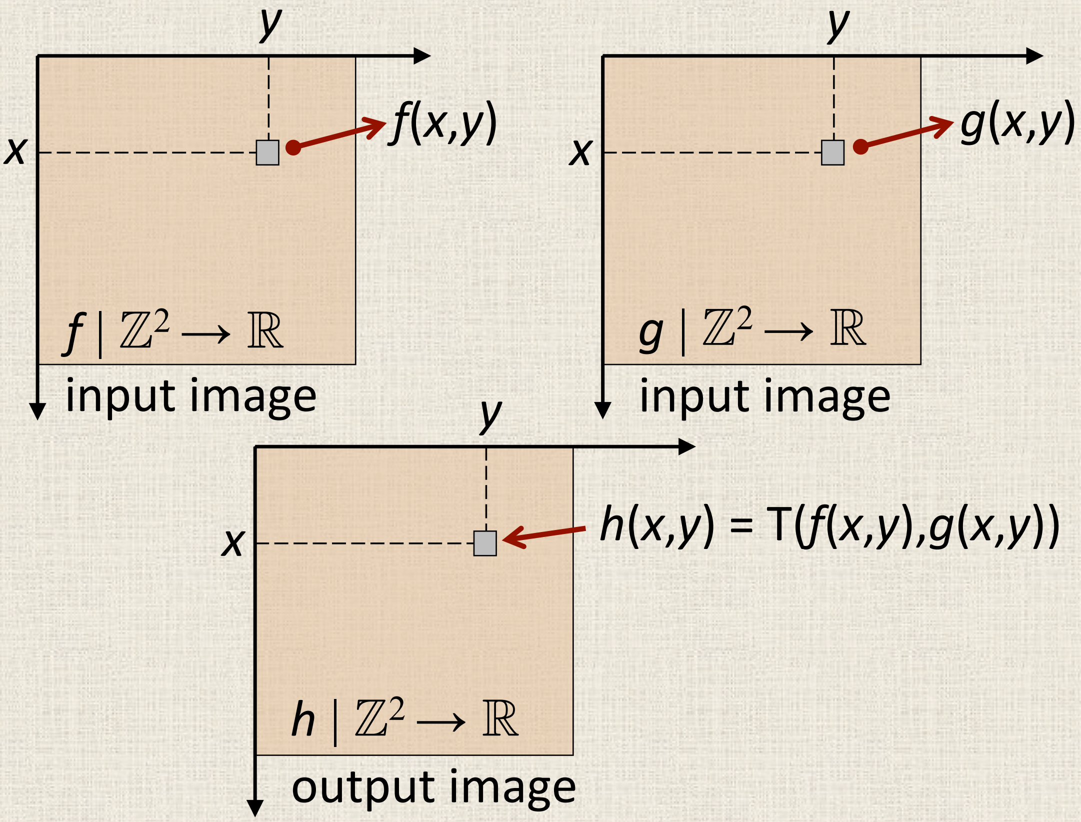
IV. Binary Single-Pixel Operations

IV.1. Principle



$f(x,y)$

$f \mid \mathbb{Z}^2 \to \mathbb{R}$

input image

$g(x,y)$

$g \mid \mathbb{Z}^2 \to \mathbb{R}$

input image

$h(x,y) = \mathrm{T}(f(x,y),g(x,y))$

$h \mid \mathbb{Z}^2 \to \mathbb{R}$

output image

IV.2a. Arithmetic Operations

$$h=f+g \;\left|\; \begin{array}{l} \mathbb{Z}^2 \longrightarrow \mathbb{R} \\ (x,y) \mapsto (f+g)(x,y) = f(x,y)+g(x,y) \end{array}\right.$$

$$h=fg \;\left|\; \begin{array}{l} \mathbb{Z}^2 \longrightarrow \mathbb{R} \\ (x,y) \mapsto (fg)(x,y) = f(x,y)g(x,y) \end{array}\right.$$

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IV.2b. Arithmetic Operations

f

g

$h=|f-g|$

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IV.3a. Logical Operations

$$h=f_{\wedge}g \left|\begin{array}{l} \mathbb{Z}^2 \longrightarrow \mathbb{R} \\ (x,y) \mapsto (f \wedge g)(x,y) = \min\{f(x,y),g(x,y)\} \end{array}\right.$$

$$h=f_{\vee}g \left|\begin{array}{l} \mathbb{Z}^2 \longrightarrow \mathbb{R} \\ (x,y) \mapsto (f \vee g)(x,y) = \max\{f(x,y),g(x,y)\} \end{array}\right.$$

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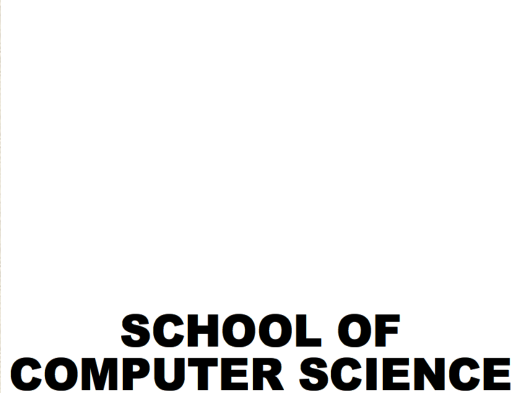
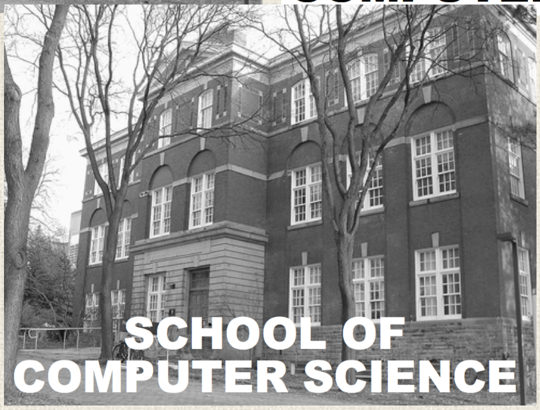
IV.3b. Logical Operations

f



g

$f \vee \neg g$



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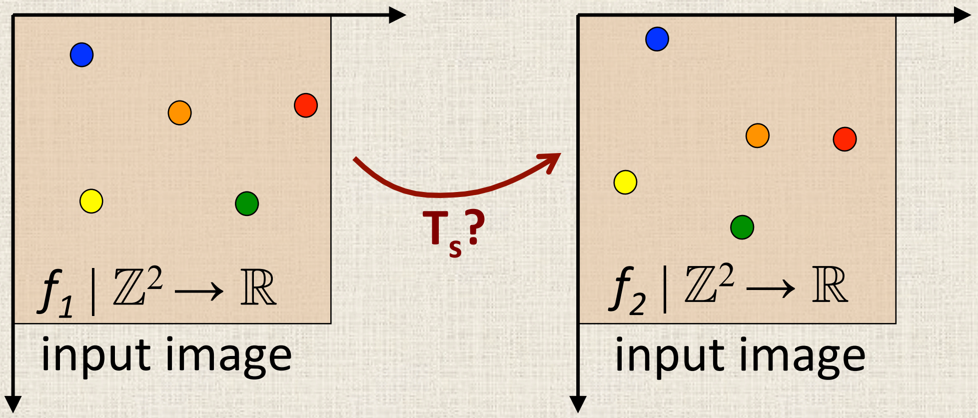
V. Other Transformations

V.1. Image Registration



$f_1 \mid \mathbb{Z}^2 \rightarrow \mathbb{R}$

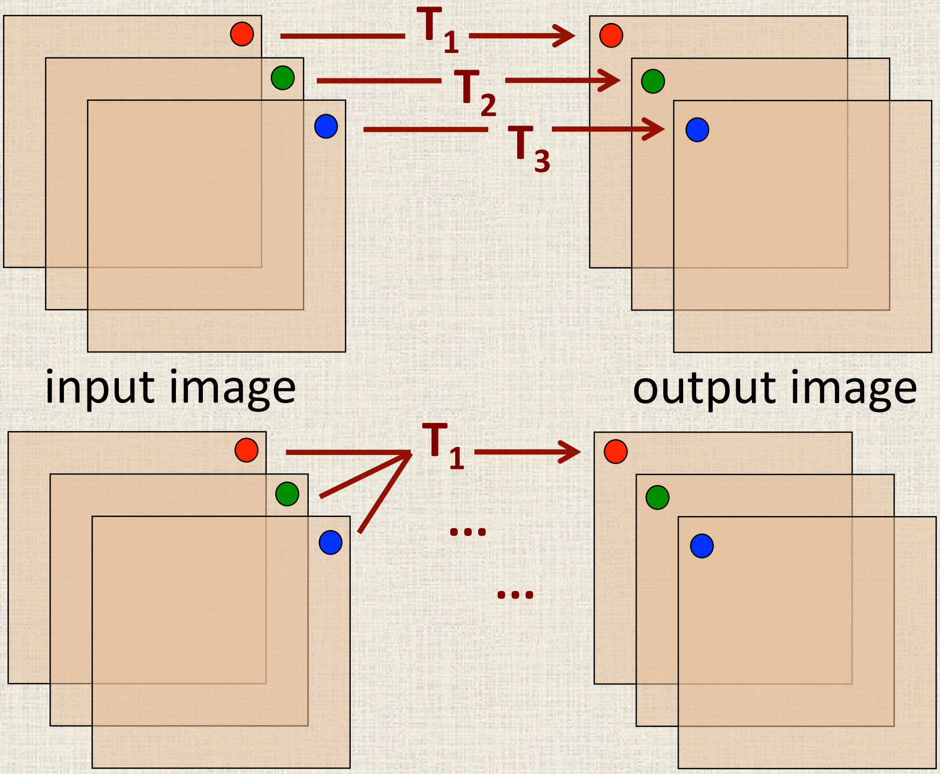
input image

T_s?

$f_2 \mid \mathbb{Z}^2 \rightarrow \mathbb{R}$

input image

V.2. Colour/Multispectral Transformations



V.3. Image Transforms