F-Histograms and Fuzzy Directional Spatial Relations

Pascal Matsakis [1], James M. Keller [1], Laurent Wendling [2]

[1] Computer Engr. & Computer Sci. Dept.
University of Missouri-Columbia
Columbia, MO 65211
USA

[2] LORIA, BP 239
54506 Vandoeuvre Cedex
FRANCE

pmatsakis@cecs.missouri.edu, keller@cecs.missouri.edu, wendling@loria.fr

Abstract

The quantitative (or fuzzy qualitative) assessment of directional spatial relationships (such as "to the right of", "above", "south of"…) between two areal objects often relies on the computation of a histogram of angles, which provides a representation of the relative position of the objects. In a recent paper, the notion of the histogram of forces was introduced. Here, we show that this powerful tool of representation lends itself, with great flexibility, to the definition of directional spatial relations. Indeed, any family of directional relations that relied on the construction of angle histograms can be advantageously redefined using force histograms. Moreover, the notion of the histogram of forces enables radically new families to be conceived: definitions which correspond to a coherent and rational perception of the world, but non-realizable by previous methods.

Keywords

Spatial relationships, parameter extraction, pattern recognition, scene analysis, fuzzy relations, fuzzy sets.

1. Introduction

Knowing how to apprehend the spatial organization of 2-D objects is essential to computer vision (for pattern recognition, image understanding, scene description in natural language, etc.). Freeman [4] proposed that the relative position of two objects be described in terms of spatial relationships. He also proposed that fuzzy relations be used, because "all-or-nothing" standard mathematical relations are clearly not suited to models of spatial relationships. Moreover, according to Bloch [2], "although the human way of reasoning can deal with qualitative information, computational approaches of spatial reasoning and object recognition can benefit from more quantitative measures" (see *e.g.* [3]). Freeman's ideas were widely adopted. But many authors assimilated 2-D objects to very elementary entities such as a point (barycenter) or a (bounding) rectangle. This process is extremely practical, therefore it has often been used, notably for spatial reasoning and representation and processing of qualitative spatial knowledge (see *e.g.* [3, 13, 14, 20]). However, a lot of morphological information on the considered objects is lost, and the procedure cannot be hoped to give a satisfactory modelling of the relationships. By introducing the notion of the histogram of angles, Miyajima and Ralescu [17] developed the idea that the relative position between two objects can have a representation of its own and can thus be described in terms other than spatial relationships. An ideal representation, once computed, is expected to allow rapid fuzzy qualitative evaluation of any spatial relationship. Actually, an angle histogram is generally used to assess the directional relationships only (such as "to the right of", "above"…) [9, 10, 12, 17, 18]. As a matter of fact, relative position is often assimilated to directional relations. The point of view is improper, but it shows the importance of these particular spatial relations —on which we focus here—in computer vision.

Finally, numerous methods for defining families of directional relations can be found in the literature. However, few of these methods simultaneously meet the following requirements:

(a) No 2-D object is assimilated to a very elementary entity such as a point or a rectangle.
(b) The defined directional relations are fuzzy relations, and not "all-or-nothing" ones.
(c) The defined family of relations satisfies the basic axiomatic properties (see Section 2.3) which are — in a more or less explicit way — widely adopted by computer scientists: for instance, we expect that object B is to the left of object A as A is to the right of B (*semantic inverse notion*, according to Freeman [4]; *symmetry property*, according to Bloch [2]).

The centroid method (see *e.g.* [9] and the methods described in [6, 8] do not meet requirement (a); the ones described in [1, 5, 11] do not meet requirement (c). Actually, as far as we are aware, the only methods which fairly meet the previous requirements are based—explicitly or not—on the notion of the histogram of angles. These methods are the compatibility method [17], the aggregation method [12], the possibility method proposed in [2] (but not the necessity method, neither the average one), and maybe the neural network methods [10].

In this paper, we show that the corresponding families of directional relations can be advantageously redefined using force histograms [15, 16] instead of angle histograms. We also introduce two new families of directional relations, still based on the notion of the histogram of forces. We thus demonstrate that this powerful tool of representation lends itself, with great flexibility, to the definition of directional spatial relations. In Section 2, different mathematical functions associated with the notion of the histogram of forces are briefly presented. In particular, it is shown how what is called an H function can participate in the generation of a family of directional relations. Actually, any method ([17, 12, 2, 10]) that fairly meets the three requirements corresponds to the datum of an H function (even though these methods are based on the notion of the histogram of angles). This point is dealt with in Section 3, and the advantages of exploiting force histograms instead of angle histograms are described. In Section 4, a new H function is introduced. We use it in Section 5 to generate two new families of directional relations. Experimental results are given in the same section.

2. Directional relations and force histograms

The Euclidean plane is referred to a directional orthogonal frame $(O, \vec{i}, \vec{j})$. Let θ and v be two reals and $\vec{i}_\theta$ and $\vec{j}_\theta$ the respective images of $\vec{i}$ and $\vec{j}$ through a θ-angle rotation: $\Delta_\theta(v)$ denotes the oriented line whose frame is defined by the vector $\vec{i}_\theta$ and the point of coordinates (0,v) — relative to $(O, \vec{i}_\theta, \vec{j}_\theta)$ — (Fig. 1).

2.1. *The μ functions*

A *fuzzy directional spatial relation between points* is a fuzzy binary relation R_α between points, where α represents any real. The relation R_α connects any couple (A,B) of distinct points with an element of interval [0,1]. This element $R_\alpha(A,B)$ is the degree of truth of the proposition "A is in direction α of B". A family $(R_\alpha)_{\alpha\in\mathbb{R}}$ of directional relations between points can be defined from a fuzzy subset of $\mathbb{R}$: its membership function μ is continuous, with period 2π, even, decreasing on [0,π], and takes

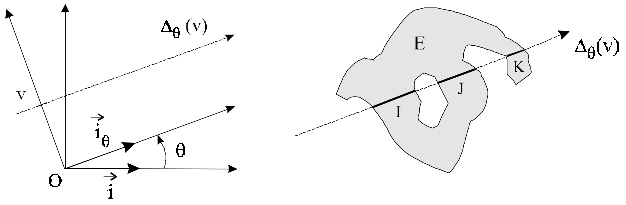


Figure 1. *Oriented straight lines and longitudinal sections.* $E \cap \Delta_\theta(v)$ (*i.e.* $I \cup J \cup K$) is a longitudinal section of E.

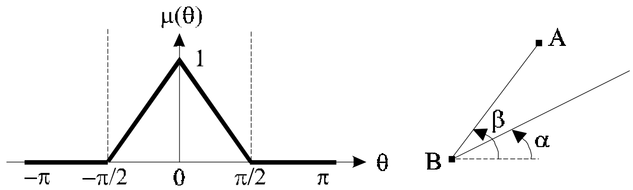


Figure 2. *Example of directional relations between points.* The degree of truth of the proposition "A is in direction α of B" is μ(β−α).

the value 1 at 0 and the value 0 at π/2. Let α and β be two real numbers and A and B be distinct points. If β is an $(\vec{i}, \vec{BA})$ angle measure, then (Fig. 2): $R_\alpha(A,B) = \mu(\beta-\alpha)$.

2.2. *The F functions*

With any couple (A,B) of 2-D objects, we associate a function F^{AB} from $\mathbb{R}$ into $\mathbb{R}_+$ [15, 16]. This function represents the relative position of A with regard to B. For any direction θ, the value $F^{AB}(\theta)$ is the total weight of the arguments that can be found in order to support the proposition "A is in direction θ of B". More precisely, it is the scalar resultant of elementary forces. These forces are exerted by the A points on those of B, and each tends to move B in direction θ. Actually, F denotes a function which allows the 2-D objects to be handled as 1-D entities (*longitudinal sections*, see Fig. 1). Let r be a real. If the elementary forces are in inverse ratio to d^r, where d represents the distance between the points considered, then F is denoted F_r. For instance, the F function associated with the universal law of gravitation is F_2. If F^{AB} is defined on IR—*i.e.* if for any θ the scalar resultant $F^{AB}(\theta)$ is finite—then the couple (A,B) is termed F-*assessable* and F^{AB} is called the *histogram of forces associated with* (A,B) *via* F, or the F-*histogram associated with* (A,B). For any real r, any couple of disjoint objects is F_r–assessable [16].

2.3. *The H functions*

A histogram of forces is an element of $Map_{2\pi}(\mathbb{R}, \mathbb{R}_+)$, *i.e.* a map from $\mathbb{R}$ into $\mathbb{R}_+$ with period 2π. For any such map h and for any real α, let us denote by h⊕α the function defined by:

$$h\oplus\alpha \mid \mathbb{R} \to \mathbb{R}_+$$
$$\theta \mapsto h(\theta+\alpha)$$

It is also an element of $Map_{2\pi}(\mathbb{R},\mathbb{R}_+)$. Now, let F be a function from T into $\mathbb{R}_+$ and H a map from $Map_{2\pi}(\mathbb{R},\mathbb{R}_+)$ into [0,1]. The family $(R\alpha)_{\alpha\in\mathbb{R}}$ of fuzzy binary relations defined on exactly the set of F-assessable couples by $R\alpha(A,B) = H(F^{AB}\oplus\alpha)$ (see Fig. 3) is called the *family of directional spatial relations generated by* F *and* H. The value $R\alpha(A,B)$ represents the degree of truth of the proposition "A is in direction α of B".

Proposition:

Let μ be the membership function of a fuzzy set "directional relations between points" (see 2.1), and H be a map from $Map_{2\pi}(\mathbb{R},\mathbb{R}_+)$ into [0,1] such that [H1] to [H3]:

[H1] For any real δ and any strictly positive real η, there exists an element ε of]0,π[such that:
$\forall h\in Map_{2\pi}(\mathbb{R},\mathbb{R}_+)$,
$\big(h\neq 0 \text{ and } h([\delta-\pi,\delta-\varepsilon]\cup[\delta+\varepsilon,\delta+\pi]) = \{0\}\big)$
$\Rightarrow |H(h)-\mu(\delta)|<\eta$

[H2] $\forall (h_1,h_2)\in Map_{2\pi}(\mathbb{R},\mathbb{R}_+)^2$,
$\big(\forall\theta\in\mathbb{R},\ h_1(-\theta) = h_2(\theta)\big) \Rightarrow H(h_1) = H(h_2)$

[H3] $\forall (h_1,h_2)\in Map_{2\pi}(\mathbb{R},\mathbb{R}_+)^2$,
$\big(\exists K\in\mathbb{R}_+^* \,/\, h_1= K.\, h_2\big) \Rightarrow H(h_1) = H(h_2)$

It is shown in [15, 16] that for any real number r, the family $(R\alpha)_{\alpha\in\mathbb{R}}$ of directional spatial relations generated by F_r and H then satisfies the following properties:

[R1] Let A and B be two objects and α and β two reals. Denote by $t_{\vec{u}}$ the translation of vector $\vec{u}$. There exists a real number k_0 such that for any k greater than k_0 the $(t_{k.\vec{i}_\beta}(A),B)$ couple is F_r-assessable. Moreover: $\lim_{k\to+\infty} R\alpha\,(t_{k.\vec{i}_\beta}(A),B) = \mu(\beta-\alpha)$

[R2] Let A and B be two objects and α a real. If (A,B) is F_r-assessable then (B,A) also is and: $R_{\alpha+\pi}(B,A) = R_\alpha(A,B)$

[R3] Let A and B be two objects, α a real and *sym* a Δβ(v)-axis orthogonal symmetry. If (A,B) is F_r-assessable then (*sym*(A),*sym*(B)) also is and: $R_{2\beta-\alpha}(sym(A),sym(B)) = R_\alpha(A,B)$

[R4] Let A and B be two objects, α a real and *dil* a dilatation with a strictly positive ratio. If (A,B) is F_r-assessable then (*dil*(A),*dil*(B)) also is and: $R_\alpha(dil(A),dil(B)) = R_\alpha(A,B)$

[R1] to [R4] are the *basic axiomatic properties* (Fig. 4). [R1] signifies that two objects can be assimilated to points if they are distant enough. [R4] means that the rel-

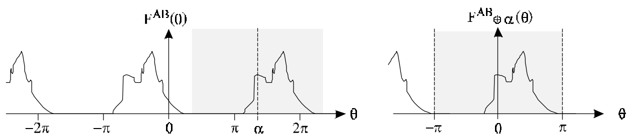
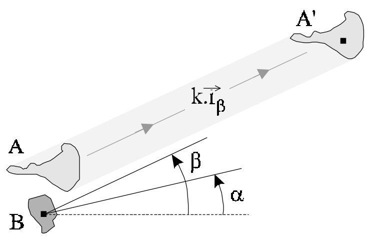
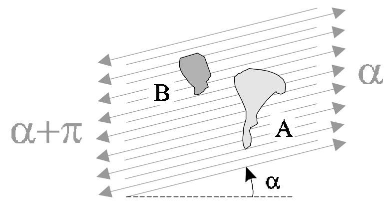


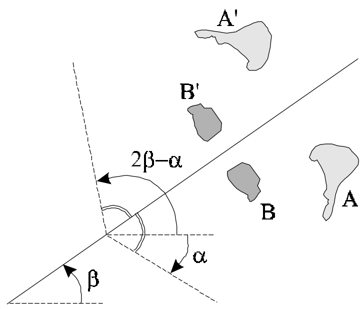
Figure 3. *Handling of force histograms:* $R\alpha(A,B) = H(F^{AB}\oplus\alpha)$



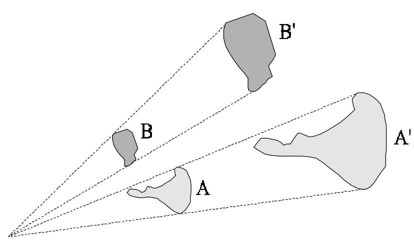
[R1] The position of A' with regard to B is approximately like the position of any point of A' with regard to any point of B.



[R2] B is in direction α+π of A as A is in direction α of B.



[R3] A' is in direction 2β−α of B' as A is in direction α of B.



[R4] The position of A' with regard to B' is like the position of A with regard to B.

Figure 4. *The basic axiomatic properties.*

ations are not sensitive to scale changes, [R3] that neither a space dimension nor a direction are preferred. [R2] brings out the notion of semantic inverse (according to Freeman [4]): object A is thus to the left of object B as B is to the right of A. The points of view stated above are — in a more or less explicit way — widely adopted by computer scientists [2, 8, 10, 11, 12, 17].

3. Directional relations and angle histograms

A and B are now two image regions. For the sake of simplicity, only disjoint crisp objects will be considered here: so, each object can be represented by a finite set of points of the Euclidean plane. The histogram of angles associated with (A,B) is a function Ang^{AB} from $\mathbb{R}$ into $\mathbb{R}_+$ with period 2π (like force histograms). For any real θ, the value $Ang^{AB}(\theta)$ is the number of couples (a,b) belonging to $A \times B$ such that θ is an $(\vec{i}, \overrightarrow{ba})$ angle measure [17, 18].

Let μ be the membership function of a fuzzy set "directional spatial relations between points" (see 2.1) and let H be a map from $Map_{2\pi}(\mathbb{R},\mathbb{R}_+)$ into [0,1] such that [H1] to [H3] (see 2.3). The family $(R\alpha)_{\alpha\in\mathbb{R}}$ of directional relations defined by $R\alpha(A,B) = H(Ang^{AB} \oplus \alpha)$ can advantageously be redefined by $R\alpha(A,B) = H(F_0^{AB} \oplus \alpha)$. Indeed, it is demonstrated in [15, 16] that F_0-histograms coincide with angle histograms, but without their weaknesses (long processing times, anisotropy, requirement for raster data, etc.). Moreover, the proposition presented in Section 2.3 vouches for the fact that the redefined family satisfies the four basic axiomatic properties. Such a guarantee *cannot* be offered to the initial family. These reflections concern the aggregation method [12], as well as the compatibility method [17] and the possibility method [2] (the neural networks methods [10] have to be kept apart: the $R\alpha(A,B)$ values depend on how exactly the training is performed).

Consider for instance the aggregation method. The μ function associated with this method is the function μ_K defined by: $\forall\theta\in[0,\pi/2]$, $\mu_K(\theta)=1-2\theta/\pi$ (Fig. 2). It is easy to show that there exists an H function H_K sharing property [H1] with μ_K, satisfying [H2] and [H3], such that the family $(R\alpha)_{\alpha\in\mathbb{R}}$ of directional relations experimented in [12] can be defined by: $R\alpha(A,B) = H_K(Ang^{AB} \oplus \alpha)$. For any element h of $Map_{2\pi}(\mathrm{IR},\mathrm{IR}_+)$ corresponding to an angle histogram, we have:

$$H_K(h) = [\Sigma_{i\in 1..n}\ h(\theta_i).\ \mu_K(\theta_i)]\ /\ \Sigma_{i\in 1..n}\ h(\theta_i)$$

where $\theta_1, \theta_2, \ldots, \theta_n$ denote the elements of $\{\theta\in]-\pi,\pi]\ /\ h(\theta) \neq 0\}$ (which is inevitably finite and non-empty). Finally, $(R\alpha)_{\alpha\in\mathbb{R}}$ can be advantageously redefined by: $R\alpha(A,B) = H_K(F_0^{AB} \oplus \alpha)$. Note that F_0-histograms are not the only histograms of forces that can be used. But in that case, redefinition obviously leads to a really different family of directional relations (in [16] for instance, three methods are compared: the first is the compatibility method; in the second, the angle histograms have been replaced by F_0-histograms; in the third, they have been replaced by F_2-histograms).

4. A new H function

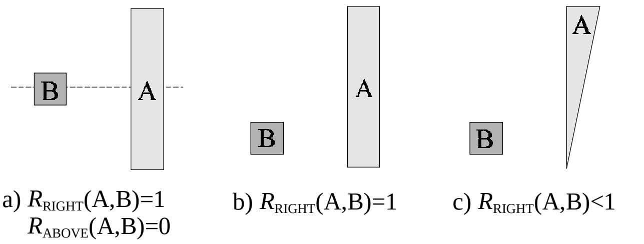


Figure 5. *Expected behavior of the directional relations.*

The H function introduced in this section accepts a physical interpretation, as the F functions do. What is to be expected from the family of directional relations generated by F and H? Four configurations, illustrated in Figure 5, are at the origin of the H function presented here. For each configuration, we have expressed a wish concerning the answer given by the directional relations. Readers are free to think this wish is arbitrary. Anyhow, arbitrariness is inherent in the problem we deal with. What matters to us is to demonstrate that the notion of the histogram of forces offers a flexible and powerful tool to define directional spatial relations.

4.1. *Contradictory, compensatory, effective forces*

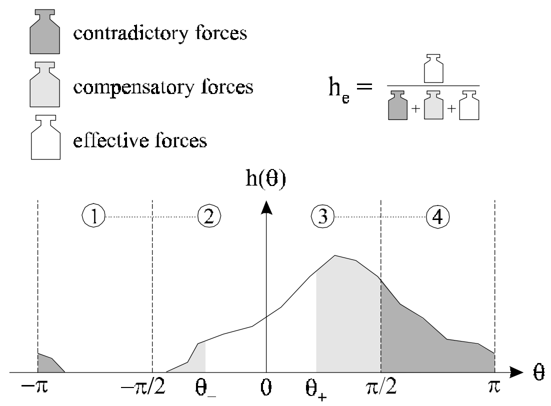


Figure 6. *Contradictory, compensatory and effective forces.*

Let (A,B) be an F-assessable couple of objects, and let h be the histogram of forces associated with (A,B) via F: $h = F^{AB}$. Any couple $(\theta,h(\theta))$, where θ is an element of $[-\pi,-\pi/2]$ (or $[-\pi/2,0]$, or $[0,\pi/2]$, or $[\pi/2,\pi]$), will be called *force of the first quadrant* (or *second*, *third*, *fourth quadrant*). The forces of the 1st and 4th quadrants are elements which, to various degrees, weaken the proposition "A is in direction 0 of B"; the forces of the 2nd and 3rd quadrants are elements which support the proposition.

The wish illustrated by Figure 5a leads us to use forces of the third quadrant in order to compensate — as much as possible — the contradictory forces of the fourth one. A proportion of these compensatory forces is defined by an element θ_+ of $[0,\pi/2]$ (Fig. 6). θ_+ is chosen such that the barycenter of the system $\{(\theta,h(\theta))\}_{\theta\in[\theta_+,\pi]}$ is— when it exists and as far as possible:

$$(\pi/2\ ,\ \int_{\theta_+}^{\pi} h(\theta).d\theta)$$

More precisely:

$$\int_{\pi}^{0} (\theta-\pi/2).h(\theta).d\theta \geq 0 \ \Rightarrow\ \int_{\pi}^{\theta_+} (\theta-\pi/2).h(\theta).d\theta = 0$$
$$\int_{\pi}^{0} (\theta-\pi/2).h(\theta).d\theta < 0 \ \Rightarrow\ \theta_+ = 0$$

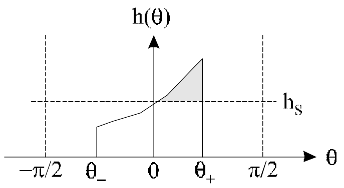
To satisfy [H2], forces of the second quadrant are used in a symmetrical way to compensate the contradictory forces of the first one. The proportion of these compensatory forces is defined by an element θ_- of $[-\pi/2,0]$ (Fig. 6):

$$\int_{-\pi}^{0} (\theta+\pi/2).h(\theta).d\theta \geq 0 \ \Rightarrow\ \int_{-\pi}^{\theta_-} (\theta+\pi/2).h(\theta).d\theta = 0$$
$$\int_{-\pi}^{0} (\theta+\pi/2).h(\theta).d\theta < 0 \ \Rightarrow\ \theta_- = 0$$

$\{(\theta,h(\theta))\}_{\theta\in[\theta_-,\theta_+]}$ is the set of the effective forces. At this stage of handling of h, the maximum value that can be reached by $R_0(A,B)$ — $R_{RIGHT}(A,B)$ — is set to the percentage h_e of these forces (Fig. 6):

$$h_e = (\ \int_{\theta_-}^{\theta_+} h(\theta).d\theta\ )\ /\ (\ \int_{-\pi}^{+\pi} h(\theta).d\theta\ )$$

4.2. *Optimal and sub-optimal components*



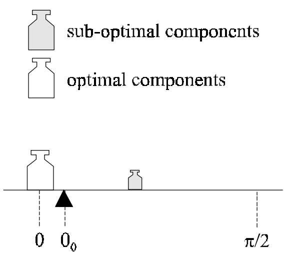


Figure 7. *Optimal and sub-optimal components.*

The wishes illustrated in Figure 5 now lead us to divide each effective force into two components (Fig. 7). These components are determined from a threshold h_S to which we will return in Section 4.3. One is optimal and used to support the idea that A is "perfectly" in direction 0 of B. The other component is sub-optimal and is used to support, more cautiously, the idea that A is "rather" in direction 0 of B. The set of sub-optimal components is assimilated to its barycenter and the set of optimal components to a unique force applying at point zero. This naturally leads us to define a representative direction θ_0 (Fig. 7) and thus to give the value of $R_0(A,B)$, *i.e.* of H(h):

$$\theta_0 = (\int_{\theta_-}^{\theta_+} \theta\,.\max(0,h(\theta)-h_S).\,d\theta\ )\ /\ (\int_{\theta_-}^{\theta_+} h(\theta).d\theta\ )$$
$$\text{and}\quad H(h) = R_0(A,B) = \mu(\theta_0).h_e$$

where μ denotes the membership function of a fuzzy set "directional spatial relations between points" (see 2.1). It is easy to show that H shares property [H1] with μ and satisfies properties [H2] and [H3].

4.3. *Directional sensitivity*

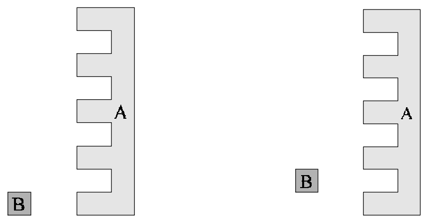


Figure 8. *The comb effect.* If h_S was set to h(0), $R_{RIGHT}(A,B)$ would take the value 1 in the left case, and could take a noticeably lower value in the right case.

Setting the threshold h_S to 0 would not allow the wish illustrated in Figure 5b to be fulfilled. Setting it to $+\infty$ would not allow 5c to be fulfilled. On the contrary, the value h(0) seems suitable. For the sake of robustness, and in order to avoid the "comb effect" (Fig. 8), it is however better to take into account the value of h not only at 0 but in a neighborhood of 0. This is the reason why we have chosen to resort to a map S from $[-\pi/2,\pi/2]$ onto $[0,1]$, which is even, continuous, decreasing on $[0,\pi/2]$, and takes the value 1 at 0 (remark that S appears in the subscript notation h_S):

$$h_S = (\int_{\theta_-}^{\theta_+} S(\theta).h(\theta).d\theta\ )\ /\ (\int_{\theta_-}^{\theta_+} S(\theta).d\theta\ )$$

S characterizes the "directional sensitivity". An analogy between S and microphone directivity—cardioid (heart-shaped), hyper cardioid, etc.—can be established: the sensitivity is maximum on the axis and decreases on moving away (Fig. 9).

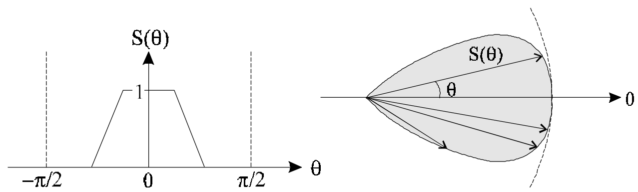


Figure 9. *Directional sensitivity.*

5. Experimental results

In this section, four families of directional spatial relations are considered: **K**, **M**, **F0** and **F2**. The first, **K**, is defined by the aggregation method [12] (see also Section 3), and the second, **M**, by the compatibility method [17]. Both are based on the construction of angle histograms. The last two families are based on the construction of force histograms. They are generated by the new H function (presented in Section 4) and by the function F_0 for **F0**, the function F_2 for **F2**. The μ and S functions used for defining H are respectively triangular (see Fig. 2) and trapezoidal (see Fig. 9). Note that the choice of S is not critical—except for some very particular configurations (the “comb effect”, see 4.3). A medium directional sensitivity has been chosen here. **K**, **M**, **F0** and **F2** can handle fuzzy objects as well as crisp objects, and **F0** and **F2** can handle vector data as well as raster data. However, the test images presented are all numerical images and involve disjoint crisp objects only: the reader can easily analyze the configurations, and the results are quite eloquent.

Let us highlight the most distinguishing mark between the **K** and **M** families on the one hand, and the **F0** and **F**2 families on the other (*i.e.*: between the existing H functions—see Section 3—and the new one—see Section 4). Consider for instance Image 7. Through a point of object B (the disc), draw a vertical line. The right half-plane so defined may contain some points of A (in white). For **K** and **M**, it is enough to conclude that the proposition “A is to the right of B” cannot be totally false: $R_{RIGHT}(A,B)\neq 0$. The **F0** and **F2** families are much more exacting. Generally, with these families, if $R\alpha(A,B)$ is not null then $R\alpha+\pi(A,B)$ is null: for instance, **F0** cannot assess A to be simultaneously quite to the left and to the right of B. On the contrary, the **K** and **M** families — especially **M** — often assess an object to be in one, and the same time, in many directions with respect to another (see Images 3, 6, 7, 8, 9). Some authors [2, 18] support the idea that this feature allows more complex relationships — like “surround” — to be derived. For instance, considering the results achieved by **M** for Image 9, one could conclude that A surrounds B. But if A was the disc and B the ring (A would then be surrounded by B), or if the ring became a disc (A would include B), **M** would achieve the same results. So, in our opinion, drawing conclusions from such results looks difficult and not reasonable. The directional relations are not the only spatial relations, and they cannot represent the relative position of an object with regard to another all by themselves. In particular, they cannot (and have not to) supply for the spatial relation “surround”. Moreover, it is well known in cognitive science that, generally, when translating visual information into natural language descriptions, people do not combine more than two relations [7, 19]. The new H function presented in Section 4, and used for defining the **F0** and **F2** families, has been conceived in that way (see Fig. 5a).

Of course, the **K** and **M** families could have been redefined using force histograms (as seen in Section 3). Finally, the notion of the histogram of forces enables all the families studied here to be conceived, and to benefit from the four basic axiomatic properties. Which family provides the “best” results? The answer obviously depends on the application considered. We just dealt here with what Gapp [6] has called the *basic meanings* of spatial relations

1 2 3

	K	M	F0	F2	K	M	F0	F2	K	M	F0	F2
RIGHT	**71**	**78**	**100**	**100**	10	12	0	0	3	7	0	0
LEFT	0	0	0	0	10	12	0	0	22	18	**75**	**99**
ABOVE	15	23	0	0	0	0	0	0	33	80	3	0
BELOW	15	23	0	0	**80**	**88**	**100**	**100**	**43**	**81**	7	2

4 5 6

	K	M	F0	F2	K	M	F0	F2	K	M	F0	F2
RIGHT	38	32	55	**86**	**60**	**71**	54	21	1	3	0	0
LEFT	0	0	0	0	2	12	0	0	40	35	**87**	**99**
ABOVE	**67**	**68**	**73**	43	0	0	0	0	17	45	0	0
BELOW	1	5	0	0	38	29	**76**	**99**	**43**	**68**	20	5

7 8 9

	K	M	F0	F2	K	M	F0	F2	K	M	F0	F2
RIGHT	1	1	0	0	4	10	0	0	**25**	**50**	0	0
LEFT	**51**	**52**	**96**	**95**	**38**	**50**	**48**	**44**	**25**	**50**	0	0
ABOVE	25	48	0	0	29	**50**	0	0	**25**	**50**	0	0
BELOW	25	48	0	0	29	**50**	0	0	**25**	**50**	0	0

Test images and result tables.
Argument A appears in white and referent B in gray.
The results are given in hundredths.

(the model proposed by Gapp to define the semantics of spatial relations distinguishes *context-specific conceptual knowledge* from the *basic meanings* of the relations). However, note that even when the results provided by **F0** and **F2** — the two new families presented in this paper — are completely different from the others, they express opinions which are fully rational (see Images 3, 4, 5, 6). Moreover, *no family* relying on the construction of angle histograms (or of F_0-histograms) can behave like **F2**: indeed, angle histograms do not take into account metric information.

Other comments:

***Image 2* —** If A is not perfectly below B in that case, when does such an event occur? The fact is that in practice **K** and **M** proscribe the equality: $R_\alpha(A,B)=1$. ***Image 3* —** According to **M** (and **K**), the "house" (object A) is rather south of the "river" (object B), or maybe north, but certainly not west. ***Image 4* — F2** is the only family to affirm that A is more to the right of B, even though it gives a certain credit to the proposition "A is above B". ***Image 5* —** As A becomes longer, **K** and **M** quickly affirm that A is essentially located to the right of B. **F0** eventually shares this point of view, but later on, and in a less definite way. **F2** is alone to maintain that A essentially remains below B. The new H function presented in Section 4 has been conceived to this end (see Fig. 5b). ***Image 7* —** According to **M**, object A is not much more to the left of object B than below or above it. And A is not much more to the left of B in Image 7 than in Image 9. ***Image 9* —** Is the ring located to the left of the disc? The **F0** and **F2** families definitely say: no. They cannot (and have not to) supply for the spatial relation "surround".

6. Conclusion

Numerous methods for defining families of directional spatial relations can be found in the literature. However, no method can be hoped to give a really satisfactory modelling of the directional relationships if it does not meet some reasonable requirements. In this paper, three requirements have been stated. It happens that the only methods which fairly meet them are based — explicitly or not — on the notion of the histogram of angles. We have shown that the corresponding families of directional relations can be advantageously redefined using force histograms instead of angle histograms. By imposing physical considerations on force histograms, we have introduced two new families of directional spatial relations. Finally, the notion of the histogram of forces offers a flexible and powerful tool to define such families. By working with two functions (F and H), endless families can be generated. Research needs to be done to determine the most suitable families according to context and to the application considered. We are currently addressing these issues.

References

[1] I. Bloch, "Fuzzy Relative Position between Objects in Images: a Morphological Approach", *ICIP'96*, Lausanne, vol. 2, pp. 987-990, 1996.

[2] I. Bloch, "Fuzzy Relative Position between Objects in Image Processing: New Definition and Properties Based on a Morphological Approach", *Int. J. of Uncertainty, Fuzziness and Knowledge-Based Systems*, submitted.

[3] S. Dutta, "Approximate Spatial Reasoning: Integrating Qualitative and Quantitative Constraints", *Int. J. of Approximate Reasoning*, vol. 5, pp. 307-331, 1991.

[4] J.Freeman, "The Modelling of Spatial Relations", *Computer Graphics & Image Processing (4)*, pp. 156-171, 1975.

[5] P. D. Gader, "Fuzzy Spatial Relations Based on Fuzzy Morphology", *Proc. of the 6th IEEE Int. Conf. on Fuzzy Systems*, vol. 2, pp. 1179-1183, 1997.

[6] K. P. Gapp, "Basic Meanings of Spatial Relations: Computation and Evaluation in 3D Space", *AAAI'94*, Seattle, WA, pp.1393-1398, 1994.

[7] K. P. Gapp, "Angle, Distance, Shape, and their Relationship to Projective Relations", *Proc. of the 17th Conf. of the Cognitive Science Society*, 1995.

[8] J. M. Keller, L. Sztandera, "Spatial Relations among Fuzzy Subsets of an Image", *Proc. of the 1st Int. Symposium on Uncertainty Modeling and Analysis*, College Park, University of Maryland, pp. 207-211, 1990.

[9] J. M. Keller, X. Wang, "Comparison of Spatial Relation Definitions in Computer Vision", *ISUMA-NAFIPS'95*, College Park MD, pp. 679-684, 1995.

[10] J. M. Keller, X. Wang, "Learning Spatial Relationships in Computer Vision", *FUZZ-IEEE*, 1996.

[11] L. T. Kòczy, "On the Description of Relative Position of Fuzzy Patterns", *Pattern Recognition Letters*, vol. 8, pp. 21-28, 1988.

[12] R. Krishnapuram, J. M. Keller, Y. Ma, "Quantitative Analysis of Properties and Spatial Relations of Fuzzy Image Regions", *IEEE Transactions on Fuzzy Systems*, vol. 1, no. 3, pp. 222-233, 1993.

[13] S. Y. Lee, F. J. Hsu, "Spatial Reasoning and Similarity Retrieval of Images Using 2D C-String Knowledge Representation", *Pattern Recognition*, vol. 25, no. 3, pp. 305-318, 1992.

[14] T. S. Levitt, D. T. Lawton, "Qualitative Navigation for Mobile Robots", *AI*, vol. 44, no. 3, pp. 305-360, 1990.

[15] P. Matsakis, *Relations spatiales structurelles et interprétation d'images*, Ph.D. dissert., Université Toulouse-III, Institut de Recherche en Informatique de Toulouse, 1998.

[16] P. Matsakis, L. Wendling, "A New Way to Represent the Relative Position between Areal Objects", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 21, no. 7, pp. 634-643, 1999.

[17] K. Miyajima, A. Ralescu, "Spatial Organization in 2D Segmented Images: Representation and Recognition of Primitive Spatial Relations", *Fuzzy Sets and Systems*, vol. 65, iss. 2/3, pp. 225-236, 1994.

[18] K. Miyajima, A. Ralescu, "Spatial Organization in 2D Segmented Images", *3rd IEEE International Conference on Fuzzy Systems*, Orlando, FL, pp. 100-105, 1994.

[19] G. Retz-Schmidt, "Various Views on Spatial Prepositions", *AI Magazine*, vol. 9, no. 2, pp. 95-105, 1988.

[20] J. Sharma, D. M. Flewelling, "Inferences from Combined Knowledge about Topology and Directions", *LNCS 951*, Springer, pp.279-291, 1995.