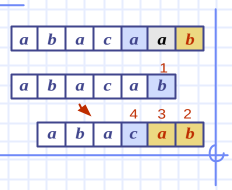
Pattern Matching



Strings

- A string is a sequence of characters
- Examples of strings:
  - C program
  - HTML document
  - DNA sequence
  - Digitized image
- An alphabet S is the set of possible characters for a family of strings
- Example of alphabets:
  - ASCII
  - Unicode
  - {0, 1}
  - {A, C, G, T}
- Let P be a string of size m
  - A substring $P[i .. j]$ of P is the subsequence of P consisting of the characters with ranks between i and j
  - A prefix of P is a substring of the type $P[0 .. i]$
  - A suffix of P is a substring of the type $P[i .. m - 1]$
- Given strings T (text) and P (pattern), the pattern matching problem consists of finding a substring of T equal to P
- Applications:
  - Text editors
  - Search engines
  - Biological research

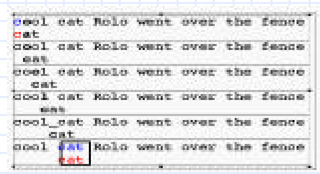
Brute-Force Algorithm

- The brute-force pattern matching algorithm compares the pattern P with the text T for each possible shift of P relative to T, until either
  - a match is found, or
  - all placements of the pattern have been tried
- Brute-force pattern matching runs in time $O(nm)$
- Example of worst case:
  - $T = aaa \ldots ah$
  - $P = aaah$
  - may occur in images and DNA sequences
  - unlikely in English text

```
function BruteForceMatch(T, P, m, n)
    Input text T of size n and pattern
        P of size m
    Output starting index of a
        substring of T equal to P or -1
        if no such substring exists
    for (i = 0; i< n ; i ++){
        /* test shift i of the pattern */
        j = 0;
        while (j < m && T[i + j] = = P[j])
            j = j + 1;
        if ( j == m)
            return i ; /* match at i */

    return -1;        /* no match */
```


Brute Force



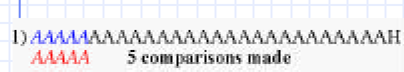
Brute Force-Complexity

- Given a pattern M characters in length, and a text N characters in length...
- Worst case: compares pattern to each substring of text of length M. For example, M=5.
- This kind of case can occur for image data.

Total number of comparisons: M (N-M+1)
Worst case time complexity: O(MN)

Brute Force-Complexity(cont.)

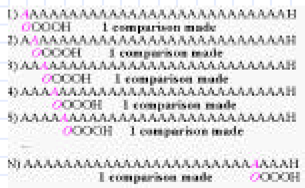
- Given a pattern M characters in length, and a text N characters in length...
- Best case if pattern found: Finds pattern in first M positions of text. For example, M=5.



Total number of comparisons: M
Best case time complexity: O(M)

Brute Force-Complexity(cont.)

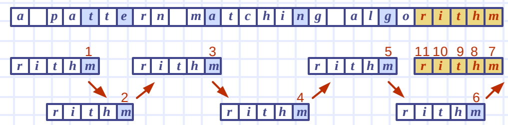
- Given a pattern M characters in length, and a text N characters in length…
- Best case if pattern not found: Always mismatch on first character. For example, M=5.



Total number of comparisons: N
Best case time complexity: O(N)

Boyer-Moore's Algorithm (1)

- The Boyer-Moore's pattern matching algorithm is based on two heuristics
  Looking-glass heuristic: Compare P with a subsequence of T moving backwards
  Character-jump heuristic: When a mismatch occurs at $T[i] = c$
  - If P contains c, shift P to align the last occurrence of c in P with $T[i]$
  - Else, shift P to align $P[0]$ with $T[i + 1]$
- Example



Last-Occurrence Function

- Boyer-Moore's algorithm preprocesses the pattern P and the alphabet S to build the last-occurrence function L mapping S to integers, where $L(c)$ is defined as
  - the largest index i such that $P[i] = c$ or
  - -1 if no such index exists
- Example:
  - $S = \{a, b, c, d\}$
  - $P = abacab$

c	a	b	c	d
$L(c)$	4	5	3	−1

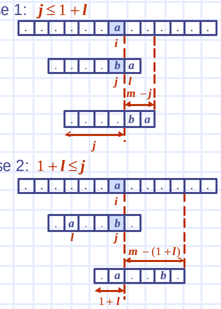
- The last-occurrence function can be represented by an array indexed by the numeric codes of the characters
- The last-occurrence function can be computed in time $O(m + s)$, where m is the size of P and s is the size of S

Boyer-Moore's Algorithm (2)

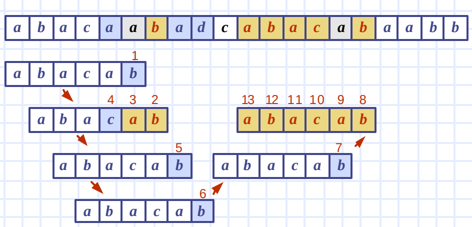
```
function BoyerMooreMatch(T, P, S )
  L = lastOccurenceFunction(P, S );
  i = m – 1;
  j = m – 1;
  repeat {
     if (T[i] == P[j])
        if (j == 0)
           return i; /* match at i */
        else {
           i --;
           j --;
        }
     else
        /* character-jump */
        l = L[T[i]];
        i = i + m – min(j, 1 + l);
        j = m – 1;
  }
  until ( i > n – 1);
  return –1;          /* no match */
```

Case 1: $j \le 1 + l$

Case 2: $1 + l \le j$

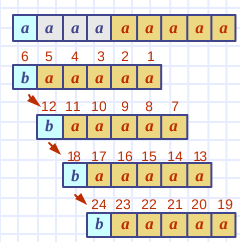


Example



Analysis

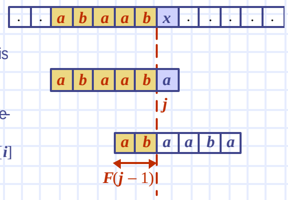
- Boyer-Moore's algorithm runs in time $O(nm + s)$
- Example of worst case:
  - $T = aaa \ldots a$
  - $P = baaa$
- The worst case may occur in images and DNA sequences but is unlikely in English text
- Boyer-Moore's algorithm is significantly faster than the brute-force algorithm on English text



KMP's Algorithm (1)

- Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- The failure function $F(i)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at $P[j] \neq T[i]$ we set $j \leftarrow F(j-1)$

j	0	1	2	3	4	5
$P[j]$	*a*	*b*	*a*	*a*	*b*	*a*
$F(j)$	0	0	1	1	2	3



KMP's Algorithm (2)

- The failure function can be represented by an array and can be computed in $O(m)$ time

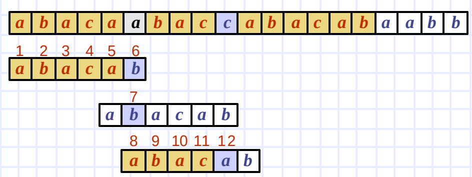
```
function FailureFunction( P)
  i = 1;
  j = 0;
  F[0] = 0;
  while (i < m){
    if (P[i] == P[j]){
      F[i ] = j + 1;
      i ++;
      j ++;
    }
    else if ( j > 0)
      j = F[j − 1];
    else{
      F[i ] = 0;
      i ++;
    }
  }
  return F ;
```

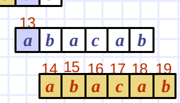

KMP's Algorithm (3)

- At each iteration of the while-loop, either
  - i increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j-1) < j$)
- Hence, there are no more than $2n$ iterations of the while-loop
- Thus, KMP's algorithm runs in optimal time $O(m + n)$

```
function KMPMatch(T, P)
  F = failureFunction(P);
  i = 0;
  j = 0;
  while (i < n){
    if (T[i] == P[j])
      if (j == m − 1)
        return (i − j ); /*match */
      else {
        i++;
        j++;
      }
    else
      if (j > 0)
        j = F[j − 1];
      else
        i++;
  }
  return −1;           /* no match */
```


Example





j	0	1	2	3	4	5
$P[j]$	*a*	*b*	*a*	*c*	*a*	*b*
$F(j)$	0	0	1	0	1	2